

Monopsony, Unemployment, and Optimal Policy

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Abstract

How does employer concentration affect unemployment, and how should unemployment insurance respond? I develop a Diamond–Mortensen–Pissarides model in which a finite number of large employers hire strategically: each firm’s hiring changes current recruiting conditions, the future unemployment pool, and rivals’ future hiring, and bargaining passes these effects into wages. In a calibrated distribution of local labor markets, replacing large employers with atomistic ones lowers unemployment from 6.0 to 5.5 percent and raises worker income excluding profits by 4.3 percent, but aggregate welfare falls slightly because atomistic multiworker firms overhire through wage dilution in bargaining. Because large employers internalize part of the congestion their vacancies create, the Hosios condition depends on market structure: efficiency requires less worker bargaining power in more concentrated markets. With risk aversion and endogenous search, whether concentrated markets call for more or less generous UI depends on the share of profits workers receive through firm ownership or profit-tax rebates.

Keywords: employer market power, search and matching, unemployment, strategic hiring, unemployment insurance.

JEL: E24, J42, J64, L13.

1 Introduction

Declining labor shares, rising firm profits, and consolidation among large employers have renewed interest in size-based labor-market power (Autor et al., 2020; De Loecker, Eeckhout, and Unger, 2020). How does the presence of a few large employers affect unemployment and workers’ welfare? Much of the literature focuses on the wages firms pay and the labor supply they face. Yet hiring has consequences beyond the wage: it changes the conditions under which every firm, including the hiring firm itself, recruits, both now and in the future. An employer that accounts for a substantial share of its labor market has reason to take these consequences into account, and so do its competitors. Employer size should matter not only for wages but for vacancy creation, job finding, and unemployment.

These are the outcomes the Diamond–Mortensen–Pissarides (DMP) model is built to study (Mortensen and Pissarides, 1994; Pissarides, 2000). The DMP model is the workhorse framework for analyzing unemployment and wage bargaining in macroeconomics. Size-based labor-market power has not been incorporated into a complete general equilibrium DMP model. I study a DMP economy with a finite number of employers who are large relative to the labor market. Each employer internalizes the influence of its employment decisions on current and future labor-market conditions, and the employment distribution becomes a strategic state variable to which it and its rivals respond. I ask how this influence changes equilibrium hiring and wage bargaining, and how it changes the appropriate design of unemployment insurance (UI). UI is increasingly discussed as part of the policy response to employer market power (Manning, 2004; Naidu and Posner, 2022), and the DMP model is the standard foundation for analyzing UI (Landais, Michailat, and Saez, 2018). Assessing UI in concentrated labor markets requires understanding size-based market power within that framework.

The paper has three main results. First, in the calibrated distribution of local labor markets, replacing large employers with atomistic employers lowers unemployment from 6 percent to 5.52 percent. Worker income excluding profits rises by 4.34 percent, while aggregate welfare falls by 0.03 percent. Many markets are already near efficient employment, and atomistic multiworker firms retain the incentive to expand employment to reduce wages within the firm. The resulting overhiring losses outweigh the gains from reducing underemployment in more concentrated markets. Second, efficiency requires a market-structure-dependent Hosios condition. A large employer already internalizes part of the congestion its vacancies create. Correcting the remaining externality requires less worker bargaining power. Third, once workers are risk averse, optimal UI depends on how concentrated markets are as well as on how profits are distributed. The share of profits reaching workers, through

ownership or redistribution, determines whether the wage increase induced by UI transfers rents from outside recipients or reallocates income within the worker population.

Two descriptive findings motivate the focus on unemployment and recruiting. In a panel of U.S. commuting zones, a 10 percent increase in a lagged local HHI is associated with about 0.08 percentage point higher unemployment, conditional on commuting-zone and year fixed effects (Appendix D). Worker flows point to the recruiting margin behind this association. In Norwegian administrative data, [Berger et al. \(2024\)](#) find that a one-standard-deviation increase in employment concentration is associated with a 0.77 percentage point lower monthly job-finding rate, against a sample mean of about 8 percent. The model links concentration to job creation through the recruiting conditions employers face.

The economy combines a finite number of employers with a common search-and-matching market. Firms produce with decreasing returns, recruit through costly vacancies, and negotiate wages under Stole–Zwiebel (1996) bargaining, which allows remaining workers to renegotiate when a worker leaves. Firms choose employment targets simultaneously as functions of the inherited employment distribution. Vacancies implement those targets at the market-clearing filling probability. A firm takes rivals’ current targets as given when choosing its own, but anticipates their future responses to the employment state it creates.

Consider a large employer deciding whether to hire one more worker. The extra hire lowers the vacancy-filling probability, raising the cost of the firm’s entire recruiting plan. A worker retained next period saves replacement hiring. Hiring today also leaves fewer workers eligible to search tomorrow, changing the recruiting conditions faced by the firm and its rivals. Rivals’ future hiring policies respond to the new employment distribution, and those responses feed back into the firm’s costs and profits. Bargaining passes these effects into wages. Better future job prospects raise workers’ outside options, and the firm’s future recruiting needs change what an extra worker is worth to it. These effects operate alongside the Stole–Zwiebel incentive in multiworker search models, as in [Elsby and Michaels \(2013\)](#) and [Acemoglu and Hawkins \(2014\)](#). With decreasing returns, an extra worker lowers marginal productivity; because existing workers can renegotiate, their wages fall too. This within-firm wage-dilution incentive remains in the atomistic benchmark. As employers become small, their individual influence on matching conditions and rivals’ future hiring vanishes.

Under monopoly, the efficiency result is simple. With linear utility and zero worker bargaining power, the employer pays the worker’s nonemployment value and bears all recruiting costs. Its hiring incentives coincide with the planner’s. At the opposite extreme, an atomistic firm ignores the congestion its vacancies impose on other recruiters. In the canonical DMP model, worker bargaining power equal to the unemployment elasticity of

matching supplies the missing private cost. Correcting matching congestion alone gives a worker bargaining weight that rises from zero under monopoly toward the matching elasticity as employers become small. The complete condition also accounts for wage-bill and strategic effects, including the within-firm bargaining distortion in the atomistic multiworker economy. In the calibration, correcting the remaining Stole–Zwiebel incentive to overhire requires worker bargaining power of 0.648 in the atomistic economy, above the matching elasticity of 0.5.

The quantitative comparison holds local markets’ relative sizes and productive capacity fixed. Across alternative calibrations, the unemployment decline from replacing large employers with atomistic ones ranges from 0.41 to 0.56 percentage point. In the Euler decomposition, own recruiting costs discourage hiring most strongly in concentrated markets, where a firm bears congestion on many of its own hires. Strategic interaction works in the other direction. When a firm expands, rivals contract their future hiring, easing recruiting pressure. The general-equilibrium (GE) wage-bill effect slightly encourages hiring under monopoly but discourages it under oligopsony. The firm’s future hiring share helps explain this difference. Its share can change under oligopsony, altering the continuation value entering wage bargaining, but is always one under monopoly.

Adding an atomistic fringe separates the size of individual employers from the share of hiring controlled by large employers together. A firm’s own hiring share determines how much of the congestion created by its recruiting falls on its own hiring plan. Employer size also enters workers’ outside options. Bargaining links the surplus from a job to the employer’s marginal recruiting cost, which increases with its hiring share. Weighting these surpluses by workers’ probabilities of joining each employer, conditional on finding a job, produces HHI. Holding matching conditions fixed, HHI summarizes the hiring-weighted recruiting contribution to the Euler equation. Wage-bill incentives and future hiring responses also depend on how employment is distributed across employers. In an equal-HHI exercise, unemployment is 5.538 percent with one large employer and 5.544 percent with two smaller employers. The difference is 0.57 basis point, or 27 percent of the one-employer market’s gap from the atomistic benchmark.

UI gives the planner another way to change hiring incentives. In the linear economy, I derive the increment to nonemployment income that implements efficient employment, accounting for congestion, the wage bill, and strategic responses. Higher benefits strengthen workers’ outside options and can correct excessive hiring. Concentrated markets with too little employment instead require lower benefits. Because a uniform policy raises or lowers

benefits in all markets, its welfare gain depends on where employment changes as well as on the total number of jobs created or lost.

To study insurance and inequality, I add risk-averse households and endogenous job search, as in the optimal-UI literature. More generous benefits protect unemployed workers against income loss but also change hiring and search. I compare optimal UI across market structures under alternative distributions of profit income, keeping the UI financing rule and welfare weights fixed. Workers can receive profits through ownership or an equal rebate of profit-tax revenue. Their share is fixed within each UI exercise.

In the calibrated economy, concentrated markets call for lower benefits when workers receive all profits. Starting from zero UI, higher benefits cost more jobs in these markets, each lost job has a higher welfare value, and the resulting wage gains come partly out of profit income shared with the unemployed. When workers receive no profits, the wage gains instead transfer income from outside recipients to workers, who value it more. Search falls more in the atomistic economy and offsets more of the upward wage pressure, so UI raises wages more in concentrated markets and the redistribution gain is larger there. A smaller worker profit share therefore raises preferred benefits, and when workers receive no profits, the monopoly–atomistic ranking reverses.

Holding the symmetric aggregate production frontier fixed, I construct a locally unique family of equilibria whose employment and wage approach the atomistic values as employers become small. The proof bounds hiring, wage, and continuation responses, establishing that the additional recruiting and wage-bill incentives disappear with firm size. For concentrated markets, I develop a numerical method that exploits employer symmetry. Each additional employer introduces another employment state, rapidly increasing the number of responses and interactions that a direct approximation must track. Many responses differ only in the identities of otherwise identical employers. I group these responses and solve for their common coefficients, retaining separate responses to changes in a firm’s own employment and in its rivals’ employment. This reduces the number of unknowns without replacing the employment distribution by an aggregate state. Policies and continuation values are approximated jointly around the symmetric steady state. The baseline computations use degree-five approximations for up to thirty employers and an independently solved atomistic benchmark.

1.1 Related literature

The paper contributes to the literature on size-based labor-market power. [Berger, Herkenhoff, and Mongey \(2022\)](#) develop a quantitative neoclassical oligopsony model in which differentiated jobs give employers upward-sloping labor supply curves. Firms internalize their influence on wages, and larger market shares generate greater wage markdowns and distort employment and production. I study size-based market power in a search-and-matching economy, where employers recruit through vacancies and their hiring decisions affect job-finding opportunities and unemployment.

An important foundation for this paper is the work of [Jarosch, Nimczik, and Sorkin \(2024\)](#) and [Berger et al. \(2024\)](#), which establishes how employer size shapes competition for workers in frictional labor markets. [Jarosch, Nimczik, and Sorkin \(2024\)](#) identify a distinct source of size-based market power: a large employer removes its own future job openings from a worker’s disagreement outside option, weakening the worker’s bargaining position. Their framework isolates the wage consequences of market structure by taking firm shares from the data and rationalizing them through vacancy costs. [Berger et al. \(2024\)](#) bring employer concentration, heterogeneous amenities, on-the-job search, and endogenous vacancy posting into a unified framework, showing how these forces jointly determine wages, worker flows, and inequality. Building on these insights, I study employers’ strategic hiring decisions. Firms internalize how their hiring changes current recruiting conditions and the future unemployment pool, together with the resulting responses of negotiated wages and competitors’ hiring.

[Gottfries and Jarosch \(2026\)](#) extend the Burdett–Mortensen wage-posting model to large employers with decreasing returns, endogenizing contact rates, firm size, and aggregate employment. Their hiring technology allows firms to generate contacts costlessly and pay a fixed cost per hire, so competition operates through wage offers and costly worker turnover without matching congestion. Here, hiring through a common DMP matching market changes the cost of the employer’s own recruiting plan and the pool of workers available for future recruitment. Employment becomes a strategic state variable in a Markov perfect equilibrium: today’s hiring changes rivals’ future hiring policies, which feed back into the firm’s recruiting costs, wages, and continuation profits.

Concurrent work also studies recruiting and labor-market power. [Bloesch, Larsen, and Yding \(2026\)](#) examine how firms combine wage offers and recruiting expenditures to attract workers, and how recruiting costs shape labor supply elasticities and the rents from monopsony. [Hurst et al. \(2025\)](#) combine directed search, monopsony power, worker heterogeneity, and capital-adjustment frictions to study the dynamic and distributional effects of

labor-market policies. Ongoing work by [Bergeaud et al. \(in progress\)](#) examines oligopsony in frictional labor markets. Here, employers change a common unemployment pool and their competitors' future hiring decisions, with consequences for bargaining, efficiency, and UI.

The bargaining framework builds on [Stole and Zwiebel \(1996\)](#) and its applications to multiworker search economies by [Elsby and Michaels \(2013\)](#) and [Acemoglu and Hawkins \(2014\)](#). With decreasing returns, an additional worker lowers marginal productivity, and renegotiation transmits this change to the wages of existing workers. The resulting within-firm wage-dilution incentive remains in my model's atomistic benchmark. Large employers also change the labor-market conditions and continuation values entering the bargain, giving them an additional influence on wages.

The efficiency analysis extends the matching-externality logic of [Hosios \(1990\)](#). Large employers bear part of the congestion cost on their own recruiting plans, changing the role of bargaining in aligning private hiring incentives with social returns. The UI application builds on general-equilibrium policy analysis in matching economies, including [Landais, Michaillat, and Saez \(2018\)](#). I study how employer concentration changes the balance between insurance, employment, and wage redistribution, and how the distribution of profit income shapes the welfare consequences of these responses.

The theoretical and numerical analysis draws on recursive methods for dynamic strategic decisions, including [Klein, Krusell, and Ríos-Rull \(2008\)](#) and the dynamic oligopoly analysis of [Bertolotti, Lanteri, and Villa \(2024\)](#). A computational contribution is to exploit employer symmetry when approximating policies and continuation values. Responses that differ only in the identities of otherwise identical employers share common coefficients. Grouping these responses substantially reduces the growth in the approximation problem as the number of firms increases, while preserving the employment distribution and the distinction between own and rival employment effects.

Section 2 presents the model. Section 3 derives hiring and wage incentives and their atomistic limit. Section 4 studies efficiency and bargaining, then introduces a competitive fringe to distinguish employer size from market concentration. Section 5 quantifies the mechanisms and their policy implications in the linear economy, including matched-size and matched-concentration comparisons with a fringe. Section 6 introduces risk aversion, endogenous search, and alternative profit-income arrangements to study optimal UI. Appendix D presents the empirical motivation and supporting evidence. The remaining appendices contain detailed derivations and proofs, numerical diagnostics, and supplementary policy and merger exercises.

2 Model

There are M employers and a unit mass of workers. Time is discrete. Employer i produces $A_i n_i^\alpha$, with $A_i > 0$ and $0 < \alpha < 1$, and output is the numeraire. Workers and firms discount at $\beta \in (0, 1)$; jobs separate at rate $\delta \in (0, 1)$. In Sections 2–5, workers have linear utility and nonemployment has flow value b . A signed transfer b_0 to each unemployed worker is the policy instrument: positive values provide UI and negative values tax nonemployment income. I use $b_0 = 0$ for the baseline allocation and introduce balanced-budget financing when evaluating policy. The transfer changes disposable income while leaving the underlying nonemployment value b fixed.

The labor-market frictions and bilateral bargaining follow the multiworker DMP framework. Employer size changes which matching conditions a firm internalizes when it chooses employment.

2.1 Timing and matching

Figure 1 summarizes the timing. The inherited state at date t is $\mathbf{n}_{t-1} = (n_{1,t-1}, \dots, n_{M,t-1})$. Write aggregate employment as $N_t = \sum_i n_{it}$. The pool eligible to search is

$$u_t = 1 - \sum_i n_{i,t-1}. \quad (1)$$

Firms simultaneously choose employment targets n_{it} . Surviving employment is $(1 - \delta)n_{i,t-1}$, so gross hiring and aggregate hiring are

$$h_{it} = n_{it} - (1 - \delta)n_{i,t-1}, \quad H_t = \sum_i h_{it}. \quad (2)$$

I study interior recruiting regions, where $h_{it} > 0$. Workers whose jobs separate enter the search pool in the following period. The current search pool therefore consists of workers who were unemployed in the preceding period.

Vacancies implement the targets through a common matching technology,

$$H_t = \mu u_t^\nu V_t^{1-\nu}, \quad V_t = \sum_i v_{it}, \quad 0 < \nu < 1. \quad (3)$$

Define tightness, the vacancy-filling probability, and the job-finding probability as

$$\theta_t = \frac{V_t}{u_t}, \quad q_t = \frac{H_t}{V_t} = \mu \theta_t^{-\nu}, \quad p_t = \frac{H_t}{u_t} = \theta_t q_t. \quad (4)$$

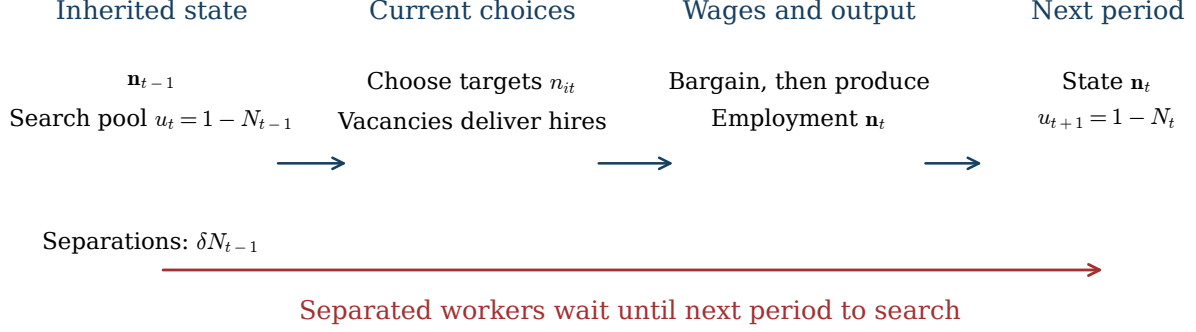


Figure 1: Employment, matching, and the search pool

Notes: The search pool is predetermined when firms choose current employment targets. Workers separating from inherited jobs wait until the next period to search. Rivals' current targets are fixed under a unilateral deviation; their future targets respond to the resulting employment state.

Hiring targets imply

$$\theta_t = \left(\frac{H_t}{\mu u_t} \right)^{1/(1-\nu)}, \quad v_{it} = \frac{h_{it}}{q_t}. \quad (5)$$

Each vacancy costs $\kappa > 0$. Admissible allocations satisfy positive search pools and hiring, feasible matching probabilities, and the no-layoff constraint. Hiring is followed by wage bargaining and production. The resulting employment vector $\mathbf{n}_t$ is the state for the next period.

The strategic variable is the employment target. Firms create the vacancies needed to implement those targets at the market-clearing filling probability. In a unilateral deviation, rivals' current targets are held fixed; the vacancy requirements that implement the targets change as q_t changes.

The current search pool is fixed when firms choose employment, so more total hiring raises tightness and lowers the filling probability. Current hiring also changes tomorrow's search pool. For a given future employment target, more inherited employment reduces both the number of new hires required and the number of workers available to fill them. These opposing changes in hiring demand and search supply determine tomorrow's filling probability.

2.2 Firms and the employment state

Let $\Pi_i(\mathbf{n}_{t-1})$ be firm value and $w_i(\mathbf{n}_t)$ the wage schedule determined by bargaining. The firm chooses its target to maximize

$$\begin{aligned}\Pi_i(\mathbf{n}_{t-1}) &= \max_{n_{it}} \{A_i n_{it}^\alpha - w_i(\mathbf{n}_t) n_{it} - \kappa v_{it} + \beta \Pi_i(\mathbf{n}_t)\}, \\ h_{it} &= n_{it} - (1 - \delta) n_{i,t-1}, \quad v_{it} = h_{it}/q_t,\end{aligned}\tag{6}$$

subject to feasibility. A Markov hiring policy is $n_{it} = \mathcal{N}_i(\mathbf{n}_{t-1})$.

The inherited employment vector and current search pool are predetermined. When firm i deviates, it takes rivals' current employment targets $\{\mathcal{N}_j(\mathbf{n}_{t-1})\}_{j \neq i}$ as given but internalizes how its choice changes total hiring, the filling probability, and its own wage schedule. Its employment today also becomes part of tomorrow's state, to which rivals' future targets respond. Thus

$$\left. \frac{dn_{jt}}{dn_{it}} \right|_{\text{current deviation}} = 0 \quad (j \neq i), \quad \frac{\partial n_{j,t+1}}{\partial n_{it}} = \frac{\partial \mathcal{N}_j(\mathbf{n}_t)}{\partial n_{it}}$$

The future policy derivative can be nonzero: today's employment changes the state governing tomorrow's hiring.

2.3 Workers and wage bargaining

Let W_{it} and U_t be the postmatching values of employment at firm i and nonemployment. In the linear benchmark,

$$W_{it} = w_{it} + \beta [(1 - \delta) W_{i,t+1} + \delta U_{t+1}],\tag{7}$$

$$U_t = b + b_0 + \beta \left[(1 - p_{t+1}) U_{t+1} + p_{t+1} \sum_j \frac{h_{j,t+1}}{H_{t+1}} W_{j,t+1} \right].\tag{8}$$

Workers search randomly, so conditional on finding a job their destination probabilities equal employers' shares of total hiring. A change in future hiring affects both the probability of finding work and the distribution of possible employers. A worker who leaves a match gives up current pay but can find another job in the future. The value of that outside option depends on how much firms will hire and on the surplus offered by the employers doing the hiring.

Wages are set by Stole–Zwiebel bargaining. The firm bargains with each worker, and the remaining workers can renegotiate if one leaves, so existing workers’ wages depend on the firm’s employment. An additional hire therefore changes the entire wage bill, and the marginal cost of employment includes that change.

Recruitment is sunk at the bargaining stage. The firm’s marginal job surplus is

$$J_{it} = \alpha A_i n_{it}^{\alpha-1} - w_i(\mathbf{n}_t) - n_{it} \frac{\partial w_i(\mathbf{n}_t)}{\partial n_{it}} + \beta \frac{\partial \Pi_i(\mathbf{n}_t)}{\partial n_{it}}. \quad (9)$$

For worker surplus $S_{it} = W_{it} - U_t$, bargaining satisfies

$$(1 - \chi)S_{it} = \chi J_{it}, \quad 0 < \chi < 1, \quad (10)$$

where χ is workers’ bargaining power. I use the regular Stole–Zwiebel wage schedule, whose payroll remains bounded as own employment tends to zero. Appendix A states this boundary condition.

2.4 Equilibrium

Definition 2.1 (Markov perfect equilibrium). A collection of hiring policies, wage schedules, firm values, and worker values is a Markov perfect equilibrium if, at every admissible state, each hiring policy solves (6) given the other employers’ policies, worker values satisfy (7)–(8), wages satisfy (10), and matching and accounting identities hold. A local equilibrium imposes these conditions and best responses on a specified invariant neighborhood of states and deviations.

Even when employers are identical, a unilateral deviation creates an asymmetric state. The conditions for a symmetric stationary equilibrium therefore depend on how wages and future hiring respond to a change in one firm’s employment, not only to changes shared by all firms.

A symmetric steady state fixes the employment levels at which the policies are evaluated. The policies also specify how firms act away from those levels. Moving a worker from one employer to another, for example, leaves aggregate employment unchanged but changes the two employers’ recruiting needs and wage bills. Their future decisions can then differ. Both the theory and the numerical solution retain the employment distribution to account for these responses.

2.5 Comparing market structures

For comparisons across symmetric market structures and for the atomistic limit, I impose

$$A_i = A_M = AM^{\alpha-1}. \quad (11)$$

Then, for a given aggregate employment N ,

$$\sum_{i=1}^M A_M (N/M)^\alpha = AN^\alpha. \quad (12)$$

This normalization holds the symmetric aggregate production frontier fixed as the number of firms changes. Comparisons across M isolate the effect of employer size relative to the labor market. The atomistic benchmark is the limit of this sequence of symmetric economies.

3 Hiring and wage determination

A large employer pays for an additional hire through its wage bill and recruiting expenditure, and gains the worker's contribution to current output and future profits. Its choice also changes matching conditions and rivals' future hiring. The first-order and envelope conditions below value these effects; wage bargaining determines how the resulting surplus is divided between the firm and its workers.

3.1 Current hiring costs

A firm needs more vacancies to raise its employment target. At fixed rival targets, the additional vacancy requirement is

$$\frac{\partial v_{it}}{\partial n_{it}} = \frac{1}{q_t} \left(1 + \frac{\nu}{1-\nu} \frac{h_{it}}{H_t} \right). \quad (13)$$

The first term is the vacancy requirement for an additional hire at the prevailing filling probability. The second is the additional vacancies needed across the firm's hiring plan when its hiring reduces that probability. Differentiating (6) gives

$$\underbrace{\alpha A_i n_{it}^{\alpha-1} + \beta \frac{\partial \Pi_i(\mathbf{n}_t)}{\partial n_{it}}}_{\text{marginal benefit}} = \underbrace{w_i(\mathbf{n}_t) + n_{it} \frac{\partial w_i(\mathbf{n}_t)}{\partial n_{it}} + \frac{\kappa}{q_t} + \frac{h_{it}}{H_t} \frac{\nu}{1-\nu} \frac{\kappa}{q_t}}_{\text{marginal cost}}. \quad (14)$$

The left-hand side values the additional worker's output and contribution to future profits. On the right, $w_i + n_{it}\partial w_i/\partial n_{it}$ is the marginal wage bill, and κ/q_t is the cost of recruiting a worker at the prevailing filling probability.

The final term is the firm's current market impact: by lowering the filling probability, an additional hire increases the vacancies needed to implement the entire hiring plan. This additional recruiting cost grows with the firm's share h_{it}/H_t of aggregate hiring. The wage bill responds to employment as well, but with the opposite sign in the canonical case. With decreasing returns, an additional worker lowers the marginal product used in bargaining, so the Stole–Zwiebel response is a saving on existing workers' wages. A large employer also changes the outside options entering that bargain. It has more hires affected by the filling probability and more workers whose wages change, so it gives these inframarginal effects more weight when choosing employment.

3.2 The continuation value of employment

The derivative of continuation profits in (14) values the state that current hiring creates. To calculate it, consider how inherited employment affects profits under firms' Markov policies. For any inherited coordinate $n_{k,t-1}$, the envelope condition is

$$\begin{aligned} \frac{\partial \Pi_i(\mathbf{n}_{t-1})}{\partial n_{k,t-1}} = & \underbrace{(1-\delta)\frac{\kappa}{q_t}\mathbf{1}\{k=i\}}_{\text{replacement hiring saved}} \\ & - \underbrace{h_{it}\frac{\nu}{1-\nu}\frac{\kappa}{q_t}\left[\frac{1}{u_t} - \frac{1-\delta}{H_t}\right]}_{\text{own future recruiting conditions}} \\ & + \underbrace{\sum_{j \neq i} \frac{\partial n_{jt}}{\partial n_{k,t-1}} \left[\beta \frac{\partial \Pi_i(\mathbf{n}_t)}{\partial n_{jt}} - n_{it} \frac{\partial w_i(\mathbf{n}_t)}{\partial n_{jt}} - h_{it} \frac{\nu}{1-\nu} \frac{\kappa}{q_t H_t} \right]}_{\text{rivals' Markov-policy responses}}. \end{aligned} \tag{15}$$

An extra inherited worker saves replacement hiring if it belongs to the firm, as recorded in the first line. The second line accounts for the change in matching conditions. Hold next period's employment targets fixed and consider an additional worker employed today. If the match survives, fewer new hires are needed to meet those targets, easing recruiting pressure. But the worker is also absent from the next search pool, making each remaining searcher more scarce. The bracket compares the changes in hiring demand and search supply. Multiplying by the firm's future hiring plan gives its gain or loss from the resulting change in the filling probability.

The last line values rivals' responses to the inherited state. A change in a rival's target affects firm i through its continuation value, wage bill, and vacancy cost. Applying this condition one period ahead in (14) gives the dynamic strategic channel. Today's choice alters tomorrow's rival policies, changing the future return to employment. The continuation derivative inside the bracket allows this feedback to recur beyond the next period.

The policy derivative measures how much a rival changes its future target; the expression in brackets gives the value of that change to the firm. A rival's contraction can ease recruiting pressure, but it also changes wages and continuation profits. Their combined effect determines whether the response encourages or discourages hiring. Section 5 measures each component in the calibrated equilibrium.

3.3 Wage determination

Consider wage bargaining at a symmetric stationary equilibrium with $n = N_M/M$. Gross hiring is δn at each firm, and

$$p = \frac{\delta N_M}{1 - N_M}, \quad \theta = \left(\frac{\delta N_M}{\mu(1 - N_M)} \right)^{1/(1-\nu)}. \quad (16)$$

Holding aggregate continuation conditions fixed gives the canonical Stole–Zwiebel slope

$$\left. \frac{\partial w_i}{\partial n_i} \right|_{SZ} = \frac{\chi \alpha (\alpha - 1) A_i}{1 - \chi(1 - \alpha)} n^{\alpha-2}. \quad (17)$$

This slope is negative because an additional worker lowers the marginal product used in bargaining. Under Stole–Zwiebel renegotiation, the resulting lower wage applies to existing workers as well as the additional worker. The firm receives a wage-bill saving when it expands. This within-firm incentive remains when the employer takes the aggregate matching environment as given.

The difference between the full own-employment derivative and (17) captures the additional wage response arising from the employer's influence on the labor market. Current employment changes future job-finding opportunities, the firm's exposure to recruiting costs, and rivals' subsequent hiring decisions. These changes affect the continuation values of workers and firms, and bargaining transmits them into the wage paid to the existing workforce.

Substituting the worker surplus and firm envelope conditions into bargaining gives

$$\begin{aligned}
w_M = & \underbrace{(1 - \chi)(b + b_0) + \chi \left[\alpha A_i n^{\alpha-1} - n \frac{\partial w_i}{\partial n_i} \Big|_{SZ} \right]}_{\text{atomistic Stole-Zwiebel component}} + \chi \beta \kappa \theta \\
& + \underbrace{\chi \beta \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}} \left[\beta \frac{\partial \Pi_i}{\partial n_j} - n \frac{\partial w_i}{\partial n_j} - \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} \right]}_{\text{strategic feedback}} \\
& - \underbrace{\chi n \left[\frac{\partial w_i}{\partial n_i} - \frac{\partial w_i}{\partial n_i} \Big|_{SZ} \right]}_{\text{GE wage-bill effect}}.
\end{aligned} \tag{18}$$

The first line is the atomistic multiworker wage evaluated at the current allocation. It combines nonemployment income, productivity corrected for within-firm wage dilution, and the value of future job-finding opportunities. The second line values the strategic continuation response from the envelope condition. The third captures the additional wage-bill response through general-equilibrium conditions.

The employment state affects today's wage through both sides of the bargain. For the worker, future tightness changes the opportunity to find another job after a separation or failed bargain. Better future opportunities raise the value of unemployment and the wage needed to sustain the current match. Hiring today affects this outside option because it changes both the number of future searchers and firms' future hiring.

The firm values future matching conditions according to how much it expects to hire. A firm with a large future recruiting plan bears substantial inframarginal costs when those conditions deteriorate. A firm expecting to fill few positions has less exposure. Current employment can change the firm's future share of total hiring and hence the continuation value entering today's bargain. Under monopoly the hiring share is always one. With several employers, a unilateral employment change can alter it even at an initially symmetric allocation.

Rivals' future policies affect the firm's continuation profits, wage bill, and recruiting costs. The strategic term in (18) passes a share of these effects into the current wage. The firm anticipates this wage response when choosing employment, so hiring and bargaining must be solved together.

The additional wage slope also changes the inframarginal wage bill and feeds back into bargaining. Appendix A gives the full derivative in (A.6), separating changes in future tightness, the firm's future hiring share, rivals' continuation responses, and wage-bill feedback.

Together, these terms make the full wage slope more or less negative than the within-firm benchmark. Expansion then saves more or less on existing workers' wages, encouraging or discouraging hiring.

3.4 Equilibrium employment

Substituting the wage equation and continuation value into the hiring condition yields

$$\begin{aligned}
0 = & \underbrace{(1 - \chi) \left[\alpha A_i n^{\alpha-1} - b - b_0 - n \frac{\partial w_i}{\partial n_i} \Big|_{SZ} \right] - \frac{\kappa}{q} + \beta(1 - \delta) \frac{\kappa}{q} - \chi \beta \kappa \theta}_{\text{atomistic multiworker benchmark}} \\
& \underbrace{- \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} + \frac{\beta}{M} (1 - \delta - p) \frac{\nu}{1 - \nu} \frac{\kappa}{q}}_{\text{current and dynamic own recruiting impact}} \\
& \underbrace{-(1 - \chi) n \left[\frac{\partial w_i}{\partial n_i} - \frac{\partial w_i}{\partial n_i} \Big|_{SZ} \right]}_{\text{GE wage-bill effect}} \\
& + \underbrace{\beta(1 - \chi) \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}} \left[\beta \frac{\partial \Pi_i}{\partial n_j} - n \frac{\partial w_i}{\partial n_j} - \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} \right]}_{\text{dynamic strategic interaction}}.
\end{aligned} \tag{19}$$

All derivatives are evaluated at the symmetric state, with their timing inherited from (15). The first line is the atomistic multiworker benchmark, including the familiar Stole–Zwiebel wage-dilution incentive. The second combines the firm's current recruiting impact with its own future recruiting conditions. The third collects the additional effect on the marginal wage bill. The fourth values rivals' future policy responses through their effects on recruiting costs, wages, and continuation profits. A positive contribution raises the marginal return to hiring at a fixed allocation.

Rivals' future responses enter the fourth line directly and also influence the wage schedule in the third. All four contributions are evaluated in the same equilibrium, with wages and policies determined jointly.

The current and dynamic own recruiting effects combine into a negative contribution:

$$T_M = -\frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} [1 - \beta(1 - \delta - p)] < 0. \tag{20}$$

The firm pays the current recruiting cost immediately. The saving in next period's hiring is discounted and occurs only if the match survives. A smaller future search pool adds a

further cost, so the combined contribution in (20) is negative. Its magnitude depends on both the hiring share $1/M$ and the equilibrium filling probability.

The wage and strategic terms depend on equilibrium responses. If the full wage slope is less negative than the canonical slope, the GE wage-bill effect lowers the return to hiring. Rivals' future responses affect that return through wage costs, recruiting costs, and continuation profits. Appendix A solves the cross-envelope recursion in (A.8). The numerical analysis evaluates all four lines of (19) jointly.

3.5 The atomistic limit

Under the production normalization (11), employers' influence on aggregate recruiting and continuation conditions vanishes as they become small. Within-firm wage dilution remains. Setting the additional market-impact terms to zero gives the atomistic employment equation

$$F_\infty(N) = \frac{(1-\chi)\alpha A}{1-\chi(1-\alpha)} N^{\alpha-1} - (1-\chi)(b+b_0) - [1-\beta(1-\delta)] \frac{\kappa}{q(N)} - \chi\beta\kappa\theta(N) = 0, \quad (21)$$

where $q(N)$ and $\theta(N)$ follow (16). The marginal-product term is decreasing in N , while both recruiting cost and tightness increase. Hence F_∞ is strictly decreasing. Its algebraic limits have opposite signs as N tends to zero and one, so it has a unique root. The maintained feasibility conditions require that this root lie in the economically admissible interior.

The associated wage is

$$w^* = (1-\chi)(b+b_0) + \frac{\chi\alpha A}{1-\chi(1-\alpha)} (N^*)^{\alpha-1} + \chi\beta\kappa\theta(N^*). \quad (22)$$

Each atomistic employer takes the matching environment as given, while its own diminishing marginal product continues to affect bargaining. The benchmark retains a within-firm hiring incentive alongside the standard matching incentives.

Convergence. The finite-employer economy approaches this benchmark when the additional wage and policy responses remain bounded as individual employment shares shrink. The following result establishes that behavior under strict feasibility, a locally stable atomistic path, and regularity of the equilibrium equations. Assumptions 1–2 in Appendix B state the conditions.

Proposition 1 (Local convergence). *Under Assumptions 1–2 and normalization (11), there is, for all sufficiently large M , a locally unique regular symmetric equilibrium near the*

atomistic economy. Its symmetric steady-state employment and wage converge to N^* and w^* . In particular, for a constant C independent of M ,

$$|N_M - N^*| + |w_M - w^*| \leq C/M. \quad (23)$$

The additional recruiting, strategic, and general-equilibrium wage contributions to (19) vanish at the same order. On the maintained invariant neighborhood, each constructed target is a strict best response to rivals' policies.

As a firm's workforce and recruiting plan shrink, fewer workers and hires are exposed to its influence on market conditions. The proof first solves the atomistic equilibrium path, then constructs nearby wage, continuation, and hiring policies for small employer shares. Along this family, the responses multiplying those shares remain bounded, so the additional incentive to influence the market vanishes with firm size.

Splitting production among more employers shrinks each firm's employment and recruiting shares, while the productivity adjustment in (11) holds the symmetric aggregate production frontier fixed. Within-firm wage dilution survives even after each firm's influence on aggregate conditions disappears, because diminishing marginal productivity still shapes bargaining inside each firm.

Appendix B provides the proof and the additional condition for extending best responses beyond the local neighborhood. The next section compares these private hiring incentives with the planner's and derives the role of bargaining and UI in efficient employment.

4 Efficiency and policy

Employer size changes both how much recruiting congestion a firm internalizes and how hiring affects its wage bill and future competitors. The planner's problem provides a common benchmark for these incentives. Comparing it with the private employment condition yields the efficient bargaining weight and the UI increment that implements the planner's employment target.

4.1 The planner's problem

With linear utility and equal weights on all income recipients, wages and profits are transfers. Aggregate welfare in a period is

$$R_t = \sum_i A_i n_{it}^\alpha + b \left(1 - \sum_i n_{it} \right) - \kappa V_t. \quad (24)$$

UI is also a transfer under balanced-budget financing and does not enter (24) directly. The planner chooses the employment vector subject to the same matching technology and timing as private firms:

$$\mathcal{P}(\mathbf{n}_{t-1}) = \max_{\mathbf{n}_t} \{ R_t + \beta \mathcal{P}(\mathbf{n}_t) \}. \quad (25)$$

At an interior allocation its Euler equation is

$$0 = \alpha A_i n_{it}^{\alpha-1} - b - \frac{\kappa}{(1-\nu)q_t} + \beta(1-\delta) \frac{\kappa}{(1-\nu)q_{t+1}} - \beta \frac{\nu}{1-\nu} \kappa \theta_{t+1}. \quad (26)$$

Efficient employment balances the value of output against forgone nonemployment and the full cost of recruiting, including its effect on future search conditions.

The planner values the same intertemporal matching effects as a private employer, but across all recruiting firms. An additional job raises current vacancy requirements, saves some replacement hiring next period, and reduces the future search pool. The social recruiting cost includes congestion on every vacancy. By comparison, a private employer directly bears congestion only on its own recruiting plan. Wage bargaining supplies an additional private cost through the surplus paid to workers. Efficiency depends on how these two private costs compare with the social cost.

4.2 Bargaining and matching congestion

An atomistic firm takes the filling probability as given. Its vacancy imposes congestion on other employers that it does not pay for. In the canonical DMP model, workers' bargaining power supplies the missing private cost: the Hosios condition sets $\chi = \nu$.

A large employer already bears part of that congestion on its own recruiting plan. Under symmetry its hiring share is $1/M$, so it internalizes that fraction of the recruiting externality. Bargaining and policy need only correct the remainder. In the pure matching comparison, this gives

$$\chi_M^{\text{congestion}} = \nu \left(1 - \frac{1}{M} \right). \quad (27)$$

Larger employers require *more firm bargaining power* to correct the recruiting externality. With a monopoly and linear utility, $\chi = 0$ and $b_0 = 0$ implement the planner: the sole employer internalizes all recruiting congestion and pays the worker's nonemployment value.

Monopoly. The sole employer's vacancies are all the vacancies in the market, so its recruiting expenditure equals the total recruiting expenditure in the planner's objective. At zero worker bargaining power and zero UI increment, the wage is b and the wage schedule is flat. Its flow profit is output less recruiting cost and bN . Adding the constant b gives aggregate welfare in (24). The firm and planner rank employment paths in the same way. Positive worker bargaining power changes the wage schedule and the firm's return to hiring.

Atomistic multiworker firms. At the opposite extreme, each employer's share of recruiting approaches zero. The matching-only comparison then requires $\chi = \nu$. The employer still bargains with an existing workforce, however, and expanding employment lowers the marginal productivity used in those bargains. The associated wage-bill saving raises its return to expansion. A bargaining weight that corrects matching congestion alone therefore leaves the within-firm hiring incentive.

Between monopoly and the atomistic limit, hiring also changes negotiated wages through aggregate conditions and rivals' future responses. Efficient bargaining must account for these effects together with matching congestion and Stole–Zwiebel dilution.

4.3 Implementing efficient employment

The complete implementation condition adds wage-bill and strategic effects to the congestion correction. For a symmetric stationary allocation, define the positive matching term

$$P_M = \frac{\nu}{1-\nu} \left\{ \frac{\kappa}{q} [1 - \beta(1-\delta)] + \beta\kappa\theta \right\}. \quad (28)$$

Let the remaining wage and strategic contribution, expressed in output units, be

$$\begin{aligned} G_M^r = & -n \frac{\partial w_i}{\partial n_i} \\ & + \beta \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}} \left[\beta \frac{\partial \Pi_i}{\partial n_j} - n \frac{\partial w_i}{\partial n_j} - \frac{1}{M} \frac{\nu}{1-\nu} \frac{\kappa}{q} \right]. \end{aligned} \quad (29)$$

The first term includes the full wage-bill effect, including Stole–Zwiebel dilution. Let F_M^D denote the right-hand side of (19) and F_M^P that of (26), evaluated at a symmetric stationary

allocation. Their difference, after scaling the planner condition by $1 - \chi$, is

$$F_M^D - (1 - \chi)F_M^P = \frac{P_M}{\nu} \left[\nu \left(1 - \frac{1}{M} \right) - \chi \right] + (1 - \chi)G_M^r - (1 - \chi)b_0. \quad (30)$$

Appendix A derives this identity term by term.

Proposition 2 (Stationary implementation). *At an interior symmetric planner steady state, the decentralized stationary Euler condition holds if and only if*

$$b_{0,M}^P = \underbrace{\frac{P_M/\nu}{1 - \chi} \left[\nu \left(1 - \frac{1}{M} \right) - \chi \right]}_{\text{remaining matching distortion}} + \underbrace{G_M^r}_{\text{wage-bill and strategic effects}}. \quad (31)$$

With $b_0 = 0$, an implementing bargaining weight, if admissible and with $P_M/\nu + G_M^r \neq 0$, satisfies

$$\chi_M = \nu \left(1 - \frac{1}{M} \right) + \left[1 - \nu \left(1 - \frac{1}{M} \right) \right] \frac{G_M^r}{P_M/\nu + G_M^r}. \quad (32)$$

All wage and policy derivatives are evaluated in the equilibrium associated with the candidate instrument and planner target.

At the planner target, $F_M^P = 0$, so both conditions follow by setting (30) to zero. The correction G_M^r changes with the instrument: bargaining power and UI affect the wage schedule and future hiring responses. I solve these objects jointly with the candidate policy at the planner's employment target.

The first term in (31) measures the congestion correction left after the employer's own internalization and workers' bargaining power. At symmetry, the employer directly internalizes the share $1/M$; bargaining then changes its share of the match surplus. The remaining term values the wage-bill and strategic incentives at the target allocation. For example, within-firm wage dilution can encourage hiring even when the matching-only gap is zero. An implementing UI increment must also offset that additional return to employment.

A higher b_0 raises the outside option and, other things equal, reduces hiring. In (31), a positive increment can correct excessive employment, including that caused by within-firm wage dilution in the atomistic economy. A concentrated market with too little employment may instead require a negative increment, measured relative to baseline nonemployment income b . The implementing policy depends on the employer's congestion internalization and on the wage and strategic responses evaluated at the planner target.

4.4 Large employers and a competitive fringe

In the symmetric economy, changing employer count changes both individual size and the number of strategic rivals. A competitive fringe separates these changes to clarify their effects on hiring and unemployment. Consider M large employers, each with productive capacity ω , and a continuum of atomistic employers with total capacity $c_f = 1 - M\omega > 0$. Their production functions are

$$Y_i = \omega A \left(\frac{n_i}{\omega} \right)^\alpha, \quad Y_f = c_f A \left(\frac{n_f}{c_f} \right)^\alpha, \quad (33)$$

where n_f is aggregate fringe employment. Total capacity remains one, so the efficient aggregate production frontier remains AN^α .

The state records employment at each large employer and aggregate fringe employment. Targets are simultaneous in Markov-perfect equilibrium: a large employer takes other current targets as given but changes matching conditions and, through tomorrow's state, rivals' future policies and aggregate fringe hiring. Individual fringe employers take matching conditions as given and retain within-firm Stole–Zwiebel bargaining.

Employer size and workers' outside options. Write $s_i = h_i/H$ for employer i 's hiring share and s_f for the combined fringe share. A unilateral increase in employment raises the employer's marginal recruiting cost to $\kappa[1 + \nu s_i/(1 - \nu)]/q$. Its own market impact therefore depends on its own share. The worker's outside option instead averages over possible hiring destinations. Bargaining links employment surplus at each destination to that employer's marginal recruiting cost. At an interior hiring choice, the destination-weighted surplus is

$$\sum_{i=1}^M s_i(E_i - U) + s_f(E_f - U) = \frac{\chi}{1 - \chi} \frac{\kappa}{q} \left[1 + \frac{\nu}{1 - \nu} \mathcal{H} \right], \quad \mathcal{H} = \sum_{i=1}^M s_i^2. \quad (34)$$

The share enters twice. It raises the employer's marginal recruiting cost, which bargaining links to the worker's surplus, and it gives the probability of joining that employer conditional on finding a job. Their product produces the squared shares in HHI. The fringe retains its probability weight, but its individual squared shares vanish. With common separation rates, hiring shares equal employment shares in the steady state.

Proposition 3 (Recruiting exposure and concentration). *At an interior stationary equilibrium, let T_i be the recruiting contribution to the hiring Euler equation. Throughout the paper, positive Euler contributions encourage hiring and negative contributions discourage it. For*

a large employer,

$$T_i = -\frac{\nu}{1-\nu} \frac{\kappa}{q} \left\{ \underbrace{s_i[1 - \beta(1 - \delta) + \beta p]}_{\text{own recruiting exposure}} - \underbrace{\beta \chi p(s_i - \mathcal{H})}_{\text{bargaining correction}} \right\}. \quad (35)$$

Including the fringe contribution T_f , the hiring-weighted recruiting account is

$$\bar{T} \equiv \sum_{i=1}^M s_i T_i + s_f T_f = -\frac{\nu}{1-\nu} \frac{\kappa}{q} [1 - \beta(1 - \delta) + \beta p] \mathcal{H}. \quad (36)$$

The own-exposure term in (35) combines current recruiting costs, retained-worker savings, and the smaller future unemployment pool. The correction arises when these conditions enter bargaining. With symmetric large employers, let s be individual share and $S = Ms$ their combined share. Then $\mathcal{H} = Ms^2 = sS$. Without a fringe, $S = 1$ and $s = \mathcal{H}$, so the bargaining correction cancels. With a fringe, $s - \mathcal{H} = s(1 - S) > 0$, leaving an offset to the large employer's recruiting cost. In the hiring-weighted sum, the bargaining corrections cancel across employer types. At given matching conditions, HHI then summarizes the recruiting contribution to the Euler equation, even though individual employers face different hiring incentives. Appendix G derives these identities.

Proposition 4 (Within-market wage premium). *At an interior stationary equilibrium with $0 < \chi < 1$, a large employer with hiring share $s_i > 0$ pays a higher wage than fringe employers in the same market:*

$$w_i - w_f = \frac{\chi}{1-\chi} \frac{\nu}{1-\nu} \frac{\kappa}{q} s_i [1 - \beta(1 - \delta)] > 0. \quad (37)$$

Holding matching conditions fixed, the premium is proportional to the employer's hiring share.

An additional hire raises the vacancies needed to implement a large employer's recruiting plan. At an interior hiring choice, the value of the marginal worker covers this higher marginal recruiting cost, and bargaining gives the worker a share of that surplus. Large and fringe employers face the same worker outside option, which cancels when their stationary wage equations are subtracted. Appendix G.2 gives the derivation.

Across markets, the comparison runs the other way. In the calibrated comparisons, concentration reduces hiring and job finding, lowering every worker's outside option, and

wages are lower in more concentrated markets (Table 2). Proposition 4 compares employers facing the same equilibrium outside option and recruiting from the same unemployment pool. With one large employer accounting for 10 percent of hiring, the large employer pays 0.7660 and fringe employers pay 0.7636, a premium of 0.32 percent. Both wages are below the atomistic wage of 0.7721; the fringe wage is 1.09 percent lower. The factor $1 - \beta(1 - \delta)$ converts the difference in the value of employment into a difference in the flow wage, so a modest wage premium sustains a larger difference in match surplus. This premium in wage levels is distinct from the GE wage-bill effect, which depends on how wages respond to employment.

Strategic rivals and the fringe. The fringe also separates two sources of future hiring responses. Combining the firm's hiring, envelope, and bargaining conditions gives

$$\begin{aligned}
0 = & \underbrace{(1 - \chi) \left[Y'_i(n_i) - b - b_0 - n_i \frac{\partial w_i}{\partial n_i} \Big|_{SZ} \right]}_{\text{canonical multiworker terms}} - [1 - \beta(1 - \delta)] \frac{\kappa}{q} - \beta \chi \kappa \theta \\
& + \underbrace{T_i}_{\text{recruiting}} + \underbrace{G_i}_{\text{GE wage-bill effect}} + \underbrace{R_i}_{\text{future large-rival response}} + \underbrace{F_i}_{\text{future fringe response}}.
\end{aligned} \tag{38}$$

The first line is the canonical multiworker hiring condition evaluated at the current allocation. The recruiting contribution T_i is given by (35), and $G_i = -(1 - \chi)n_i[\partial w_i/\partial n_i - (\partial w_i/\partial n_i)|_{SZ}]$ is the GE wage-bill effect.

Let $\mathcal{N}_j(\mathbf{n})$ denote large employer j 's next-period employment target and $\mathcal{N}_f(\mathbf{n})$ aggregate fringe employment next period, as functions of today's employment state. Evaluating derivatives at the stationary equilibrium, the two future-policy contributions are

$$R_i = \beta(1 - \chi) \sum_{\substack{j=1 \\ j \neq i}}^M \underbrace{\frac{\partial \mathcal{N}_j}{\partial n_i}}_{\text{future large-rival response}} \underbrace{\left[\beta \frac{\partial \Pi_i}{\partial n_j} - n_i \frac{\partial w_i}{\partial n_j} - s_i \frac{\nu}{1 - \nu} \frac{\kappa}{q} \right]}_{\text{effect on the firm's payoff}}, \tag{39}$$

and

$$F_i = \beta(1 - \chi) \underbrace{\frac{\partial \mathcal{N}_f}{\partial n_i}}_{\text{future fringe response}} \underbrace{\left[\beta \frac{\partial \Pi_i}{\partial n_f} - n_i \frac{\partial w_i}{\partial n_f} - s_i \frac{\nu}{1 - \nu} \frac{\kappa}{q} \right]}_{\text{effect on the firm's payoff}}. \tag{40}$$

Each contribution multiplies a future employment response by its value to firm i . Within each bracket, the first term is the change in discounted continuation profits, the second is the change in its wage bill, and the third is the additional recruiting cost. Higher employment

elsewhere lowers the filling probability and so raises the vacancies the firm needs for its own hiring plan. That cost scales with the firm’s own hiring share, which is why s_i appears in both equations whether the change comes from a large rival or the fringe.

R_i values other large employers’ future hiring responses. F_i values the aggregate response of competitive fringe employers. No fringe employer responds strategically, since each takes aggregate conditions as given, but fringe hiring as a whole still changes with the labor-market state, and a large employer anticipates that. With one large employer and a fringe, $R_i = 0$ because there is no other large employer whose policy can respond, but F_i remains, because today’s employment changes the conditions under which the fringe will recruit tomorrow. With several large employers, both terms enter.

Future policies also shape the wage schedule, and so enter G_i ; R_i and F_i are the terms in which those policies appear explicitly in the hiring condition. Appendix G.2 derives (39)–(40) and solves the recursion through subsequent hiring responses. Table 6 reports these contributions separately.

Small employers and the atomistic benchmark. Holding M fixed and reducing ω shrinks employers without adding strategic rivals. Along the regular equilibrium branch described in Appendix G, their additional recruiting and wage-bill effects vanish with individual size. Ordinary within-firm wage dilution remains: the relevant disappearing term is $n_i[\partial w_i/\partial n_i - (\partial w_i/\partial n_i)|_{SZ}]$, not the entire employment derivative of the wage bill. Explicit large-rival feedback becomes small more quickly: the employer must affect a rival’s future policy, and that response must affect its own payoff. The aggregate employment effect also depends on the fraction of hiring exposed to these large-employer incentives.

The next section quantifies these mechanisms in the linear economy. Section 6 then introduces insurance and redistribution, which change the welfare value of the wage and employment responses.

5 Quantitative analysis

The calibrated linear-utility economy contains markets with different employer counts and matches aggregate unemployment and market tightness. Holding its parameters and productive capacity fixed, I compare employment, wages, and welfare across market structures. The Euler and wage decompositions explain the hiring incentives behind these outcomes. The policy exercises then ask how bargaining power and UI can improve employment in each market.

5.1 Baseline calibration

The model is calibrated at a weekly frequency using conventional production, matching, and bargaining parameters. Table 1 reports the baseline values. The nonemployment value $b = 0.4$ follows Shimer (2005), and I set worker bargaining power and the unemployment elasticity of matching to $\chi = \nu = 0.5$.

Table 1: Baseline parameters and targets

Parameter	Value	Role or target
β	0.999	Weekly discount factor
δ	0.0048	Weekly separation probability
α	0.64	Production curvature
ν	0.50	Unemployment elasticity of matching
χ	0.50	Worker bargaining power
b	0.40	Nonemployment value
A	1	Aggregate productivity normalization
κ	0.4090	Jointly fitted recruiting cost
μ	0.08913	Jointly fitted matching efficiency
Aggregate unemployment	0.060	Calibration target
Vacancies / unemployment	0.720	Aggregate calibration target

Notes: The productivity normalization is $A_M = AM^{\alpha-1}$. The two fitted parameters jointly match the aggregate targets in the heterogeneous-market calibration.

The heterogeneous economy combines symmetric labor markets with different employer counts. I map the concentration bins in Berger, Herkenhoff, and Mongey (2022) to the symmetric identity $HHI = 1/M$, with concentration expressed on a zero-to-one scale. Using the lower endpoint of each bin gives market types $M \in \{\infty, 20, 10, 4, 2, 1, 1\}$.

Market frequencies and payroll shares are different weights. Let π_g^m denote the frequency of markets in bin g , π_g^p its payroll share, and L_g its relative labor-force size. Consistency with the model's payroll per worker implies

$$L_g \propto \frac{\pi_g^p / \pi_g^m}{N_g w_g}, \quad \omega_g = \frac{\pi_g^m L_g}{\sum_h \pi_h^m L_h}. \quad (41)$$

The fitted parameters solve

$$\sum_g \omega_g (1 - N_g) = 0.06, \quad \frac{\sum_g \omega_g V_g}{\sum_g \omega_g (1 - N_g)} = 0.72. \quad (42)$$

Because payroll depends on equilibrium employment and wages, I fit the parameters and market weights jointly. Counterfactuals hold these calibrated weights fixed.

The matching elasticity ν governs how much an increase in hiring lowers the filling probability, and the recruiting cost κ scales the resulting expenditure. The separation probability δ determines replacement hiring and how long an extra job lasts. Production curvature α governs the decline in marginal product and the within-firm wage-dilution incentive. Bargaining power χ determines how these effects enter wages and how much congestion remains uncorrected. Conditional on these parameters, the unemployment and tightness targets jointly determine μ and κ . The variation in unemployment, wages, and welfare across concentration levels is then a prediction of the model.

5.2 Solving concentrated markets

To choose current hiring, a firm needs to know how its wage and rivals' future policies respond to employment, and what those responses mean for continuation profits. I solve the hiring policy, wage schedule, and cross-envelope equations jointly in a neighborhood of the stationary allocation.

A direct polynomial approximation in M labeled state variables would quickly become large. A firm's policy, however, is invariant to relabeling its otherwise identical rivals. Exploiting this symmetry reduces the number of coefficients while keeping the firm's own employment separate. At second order, for example, its approximation includes distinct coefficients on

$$(n_i - \bar{n})^2, \quad (n_i - \bar{n}) \sum_{j \neq i} (n_j - \bar{n}), \quad \sum_{j \neq i} (n_j - \bar{n})^2, \quad \sum_{j < k; j, k \neq i} (n_j - \bar{n})(n_k - \bar{n}).$$

The last two expressions capture distinct effects: a repeated change in one rival's employment and joint changes in the employment of two different rivals. Each has its own coefficient.

The continuation derivative $\partial \Pi_i / \partial n_j$ treats two employers separately: firm i and the named rival j . The remaining firms can be relabeled. I use a separate two-employer representation for this object, retaining repeated and distinct responses among the remaining employers. Each market's approximation includes the derivative types supported by its employer count.

At a chosen polynomial degree, I match the levels and all admissible derivatives of the firm, wage, and cross-envelope residuals, solving for steady-state employment and wages together with the policy and value derivatives. At degree five the system has 12 unknowns under monopoly, 63 under duopoly, and 162 with four firms. Once there are enough employers

to represent every degree-five derivative type, it has 186 unknowns, independent of further increases in M .

I use continuation in parameters or market structure to obtain starting values, then solve the complete equations at each market structure. Appendix C reports numerical diagnostics, local stability, and approximation-order comparisons.

The baseline comparisons use complete degree-five solutions for $M \leq 30$. The common-policy exercise in Appendix C.7 uses degree three and includes an approximation-order comparison. I solve the atomistic benchmark directly from its equilibrium equations.

5.3 Employment and wages

Figure 2 and Table 2 report the allocations at $b_0 = 0$. Employment and wages rise as the number of employers grows. The largest employment changes occur among concentrated markets, where each firm accounts for a substantial share of hiring and strongly influences the future unemployment pool.

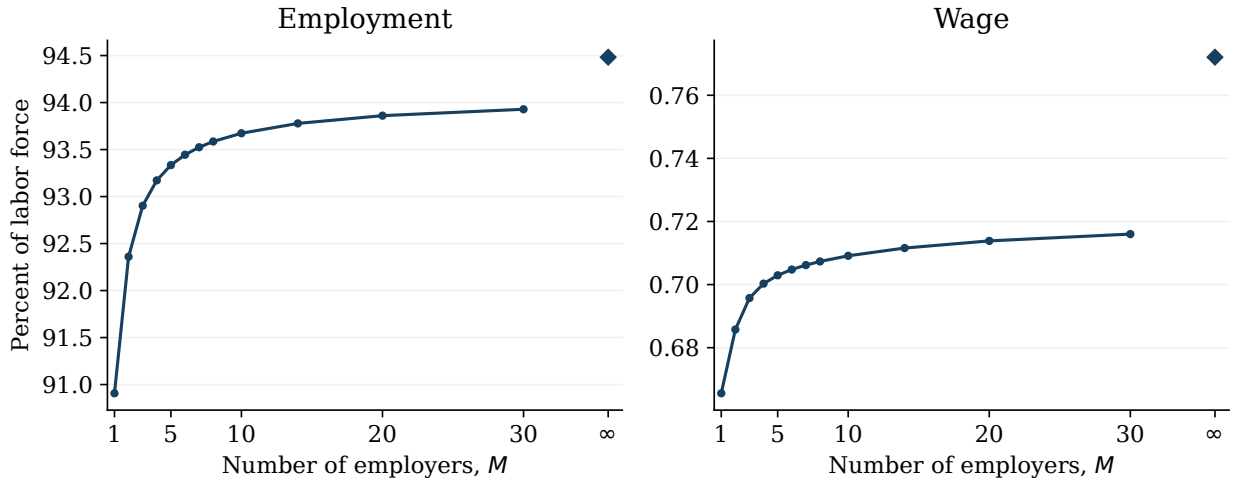


Figure 2: Employment and wages across market structures

Notes: Finite points are equilibria at $M = 1, 2, 3, 4, 5, 6, 7, 8, 10, 14, 20, 30$. Diamonds denote the atomistic benchmark. Productivity is normalized by (11), and $b_0 = 0$.

Under monopoly, unemployment is 9.09 percent. It falls to 7.64 percent with two employers and 6.07 percent with thirty employers, compared with 5.52 percent at the exact atomistic benchmark. The wage rises from 0.666 under monopoly to 0.716 with thirty employers and is 0.772 in the atomistic economy.

In the heterogeneous calibration, replacing every market by its atomistic counterpart lowers unemployment from 6 percent to 5.52 percent, a decline of about 0.48 percentage point.

Table 2: Unemployment, wages, aggregate welfare, and worker income

M	Unemployment (%)	Wage	Aggregate welfare (% vs. planner)	Worker income excluding profits
1	9.093	0.6655	−0.1625	0.6414
2	7.640	0.6858	−0.0261	0.6639
4	6.828	0.7003	+0.0002	0.6798
10	6.327	0.7091	−0.0089	0.6896
20	6.139	0.7138	−0.0184	0.6946
30	6.071	0.7160	−0.0228	0.6968
∞	5.517	0.7721	−0.0795	0.7515
Dynamic planner (steady state)	6.870	—	0.0000	—

Notes: The aggregate-welfare column reports $100(R_M/R^P - 1)$, where $R_M = AN_M^\alpha + b(1 - N_M) - \kappa V_M$ and R^P is the welfare flow at the dynamic planner’s steady state. Deviations are computed from unrounded values. The planner maximizes discounted welfare in (25); its steady state need not maximize the stationary flow, explaining the small positive entry at $M = 4$ (footnote 1). Worker income excluding profits is $Nw + b(1 - N)$ at zero UI and also measures workers’ flow welfare under linear utility.

This weighted comparison is smaller than the monopoly-to-atomistic difference because the calibrated economy includes many relatively unconcentrated markets. Market membership and labor-force weights are held fixed.

Lower unemployment comes from faster job finding. The weekly job-finding probability rises from 4.80 percent under monopoly to 6.55 percent with four employers and 8.22 percent at the atomistic limit. Over the same comparison, the vacancy-filling probability falls from 16.55 percent to 12.13 percent and 9.66 percent. More competitive markets create more hiring opportunities for workers and make each vacancy harder to fill. Appendix C reports the full set of probabilities. The separation rate remains fixed at 0.48 percent per week.

5.4 Aggregate welfare and worker income

Higher employment and wages raise workers’ income, but their implications for aggregate welfare differ. A higher wage reallocates income from profits to labor. Additional employment increases output, while requiring more recruiting expenditure and reducing the value generated outside employment. With linear utility and equal social weights, the change in aggregate welfare is

$$\Delta R = \Delta(AN^\alpha) - b \Delta N - \kappa \Delta V. \quad (43)$$

Worker income excluding profits is $Nw + b(1 - N)$; under linear utility, it also measures workers’ flow welfare. Workers gain from higher wages even when lower profits leave total

income unchanged. Aggregate welfare instead counts additional output net of recruiting costs and forgone nonemployment value.

In Figure 3, aggregate welfare first rises and then falls with the number of employers and is highest around four employers among the displayed finite points. The difference between its highest and lowest values is 0.163 percent of the dynamic planner’s stationary welfare. Worker income excluding profits rises by about 17.2 percent from monopoly to the atomistic benchmark, compared with 4.34 percent in the calibrated distribution of markets.

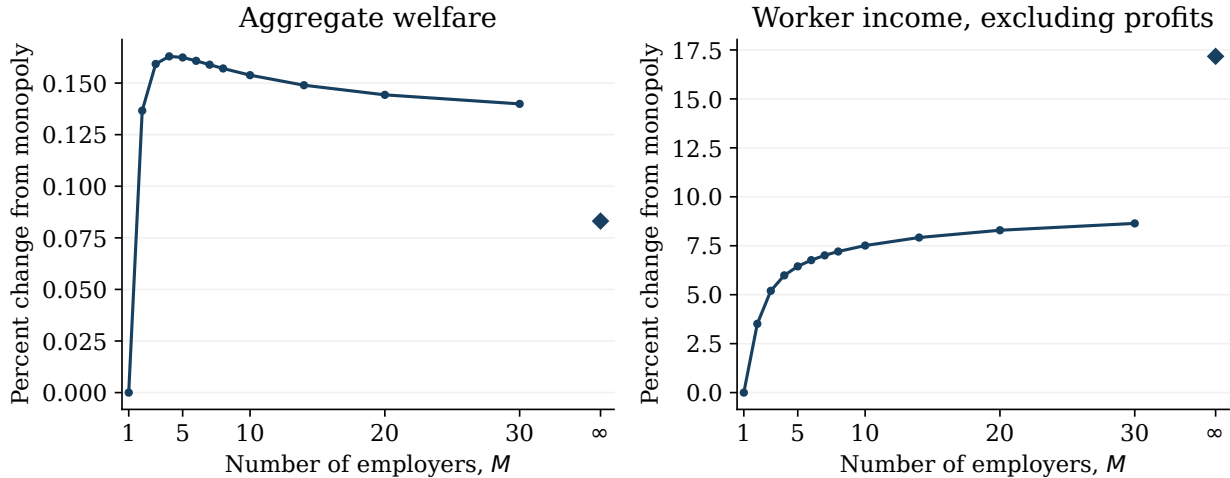


Figure 3: Aggregate welfare and the incidence on workers

Notes: Each panel is measured as a percentage change from its own monopoly value. The panels have different vertical scales. The maximum-minus-minimum aggregate-welfare difference is 0.163 percent of the dynamic planner’s stationary welfare. Worker income excludes profits.

The welfare curve is relatively flat near the efficient allocation. Concentrated markets have too little employment relative to the dynamic planner; sufficiently unconcentrated markets have too much because of the remaining multiworker bargaining distortion. The four-employer allocation lies near the efficient region because recruiting and bargaining distortions approximately offset there.¹

Measured on the common scale in Table 3, extra output from monopoly to the atomistic economy contributes 2.434 percent of monopoly aggregate welfare. Forgone nonemployment value subtracts 1.480 percent and additional recruiting subtracts 0.871 percent, leaving a net gain of 0.083 percent. Workers gain much more, 11.398 percent on this same aggregate-welfare scale, almost entirely at the expense of profits.

¹The dynamic planner in (25) values transitions as well as the eventual stationary allocation. Its steady-state employment is 0.931300, compared with 0.932137 from directly maximizing stationary welfare $R(N)$. The corresponding stationary welfare levels are 0.96797405 and 0.96797677. The bargaining-implementation exercise targets the dynamic planner’s steady state. The common-UI exercise and Section 6 optimize stationary welfare.

Table 3: Output, recruiting costs, and the distribution of income

M	Output	Nonemployment	Recruiting	Aggregate	Workers	Profits
1	0.000	0.000	0.000	0.000	0.000	0.000
2	+0.993	-0.601	-0.255	+0.137	+2.331	-2.195
4	+1.546	-0.938	-0.445	+0.163	+3.974	-3.811
10	+1.886	-1.145	-0.587	+0.154	+4.984	-4.831
20	+2.013	-1.223	-0.646	+0.144	+5.503	-5.358
∞	+2.434	-1.480	-0.871	+0.083	+11.398	-11.315

Notes: Changes from monopoly, as percentages of monopoly aggregate welfare. The nonemployment and recruiting columns are signed contributions to welfare. Output plus nonemployment plus recruiting equals aggregate welfare; worker income plus profits gives the same total. Worker income excludes profits.

The same accounting applies to the heterogeneous-market counterfactual that delivers the headline unemployment result. Table 4 holds market sizes and productive capacity fixed and replaces each market by its atomistic counterpart. Unemployment falls by 0.483 percentage point, worker income excluding profits rises by 4.34 percent, and profits fall by 12.77 percent. Aggregate welfare falls slightly, by 0.030 percent. Additional output contributes 0.327 percent of baseline aggregate welfare, but forgone nonemployment value and additional recruiting expenditure subtract 0.200 and 0.158 percent, respectively.

Many markets in the calibrated distribution are already near efficient employment. Replacing their remaining large employers pushes employment beyond that level because atomistic employers take market-wide recruiting conditions as given but still gain from within-firm wage dilution. The resulting welfare losses outweigh the efficiency gains in more concentrated markets. Higher wages nevertheless raise workers' income by redistributing profits.

5.5 The sources of the employment response

Figure 4 separates the marginal return to hiring into the familiar multiworker DMP benchmark and three additional contributions: own recruiting costs, wage-bill effects, and strategic interaction. All terms are evaluated at the same equilibrium for each market structure.

Own recruiting costs discourage hiring at every market structure in Figure 4. A firm that expands raises the cost of its current recruiting plan and leaves fewer workers available for future recruitment. These costs exceed the discounted saving in replacement hiring. As the number of employers increases, each firm bears congestion on fewer of its own hires, and its recruiting contribution becomes smaller in magnitude.

Rivals' future responses work in the other direction. When a firm expands today, its rivals reduce their future employment targets, easing recruiting conditions. Their contraction also

Table 4: Replacing large employers in the calibrated market distribution

	Baseline	Atomistic	Change
Unemployment (percent)	6.000	5.517	−0.483 pp
Worker income excluding profits	0.7203	0.7515	+4.34%
Profits	0.2472	0.2157	−12.77%
Aggregate welfare	0.9675	0.9672	−0.030%
<i>Welfare contributions: percent of baseline aggregate welfare</i>			
Additional output			+0.327
Forgone nonemployment value			−0.200
Additional recruiting expenditure			−0.158
Net change			−0.030

Notes: Labor-force weights and structural parameters are fixed at their baseline values, calculated before rounding. Worker income is $Nw + b(1 - N)$ and excludes profits. Changes in the first panel are relative to each variable’s own baseline, except unemployment, which is in percentage points. The second panel uses one common denominator. Contributions add before rounding.

changes wages and continuation profits, but the net strategic contribution encourages hiring. This contribution is absent under monopoly, appears with a second employer, and declines as further employers are added and firms become smaller.

The GE wage-bill effect follows a different pattern. It slightly encourages hiring under monopoly, becomes contractionary with a second employer, initially grows in magnitude, and then recedes toward the atomistic benchmark. A contractionary contribution means that the wage falls less with the firm’s employment than under the canonical Stole–Zwiebel bargain. Expansion generates a smaller saving on the wages of existing workers. To understand the change from monopoly to oligopsony, consider how current employment changes the firm’s share of future hiring and the continuation value entering the wage bargain.

The current recruiting cost and the saving in future replacement hiring are individually much larger than their net contribution (Table 5, panel A). Retaining an additional worker almost offsets the current recruiting cost. The smaller future search pool then accounts for most of the remaining contractionary incentive, including about 91 percent of the net recruiting effect under duopoly. The recruiting contribution falls toward the magnitude of the GE wage-bill effect as markets become less concentrated. They are similar at ten employers, and recruiting is smaller at thirty. The within-firm Stole–Zwiebel effect remains in the benchmark component, separate from the GE wage-bill effect.

In these comparisons, adding employers also makes each one smaller. Section 5.6 uses a competitive fringe to vary their number and individual hiring shares separately.

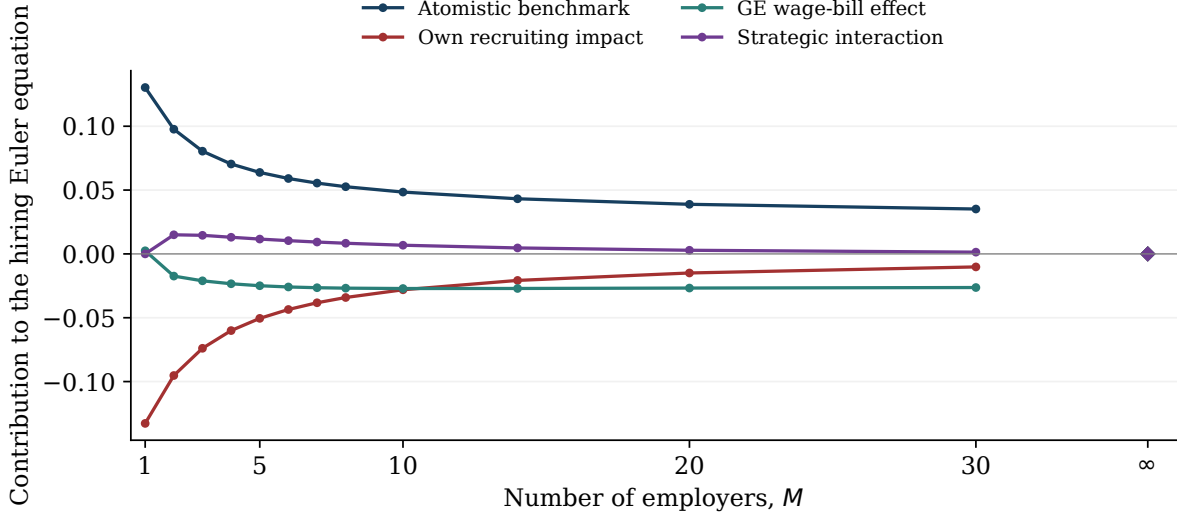


Figure 4: Decomposition of the hiring Euler equation

Notes: The four contributions are the four lines of (19), evaluated in the same zero-policy equilibrium. Positive values raise the marginal return to hiring at a fixed allocation. They sum to zero, up to numerical residuals.

Table 5: Hiring incentives under duopoly and four employers

Contribution	Two employers	Four employers
<i>A. Own recruiting</i>		
Current recruiting cost	-1.4938	-0.8432
Retained-worker savings	+1.4852	+0.8383
Future search pool, including bargaining	-0.0866	-0.0552
Net recruiting contribution	-0.0953	-0.0601
<i>B. Strategic interaction</i>		
Rivals' recruiting response	+0.0290	+0.0189
Rivals' wage response	-0.0141	-0.0058
Net strategic contribution	+0.0150	+0.0130
<i>C. GE wage-bill effect</i>		
Future tightness	+0.0079	+0.0047
Future hiring share	-0.0223	-0.0353
Rivals' continuation response	+0.0019	+0.0131
Wage-bill feedback	-0.0049	-0.0059
Net GE wage-bill effect	-0.0174	-0.0234

Notes: All entries use the signed Euler units of (19): positive values encourage hiring. Panel A decomposes the recruiting term; panel B decomposes strategic interaction; panel C decomposes the GE wage-bill effect. Components add before rounding.

5.5.1 Rival responses and future recruiting conditions

Rivals' aggregate next-period employment response is negative throughout the concentrated-market range in Figure 5. For each additional worker hired by a firm today, rivals' total employment falls by about 0.0245 workers under duopoly. With four employers, the combined response is about -0.0375 .

The right panel decomposes the associated future-tightness response. In one calculation, the firm's own future policy and the unemployment pool respond, while rivals' future targets are held fixed. In the other, rivals also follow their Markov policies. At duopoly, the elasticity of future tightness with respect to the firm's current employment is about 4.92 with rivals' targets held fixed, compared with -0.18 in equilibrium. The smaller search pool would substantially tighten recruiting conditions at those fixed rival targets. Rivals' subsequent contraction eases hiring demand and absorbs that pressure.

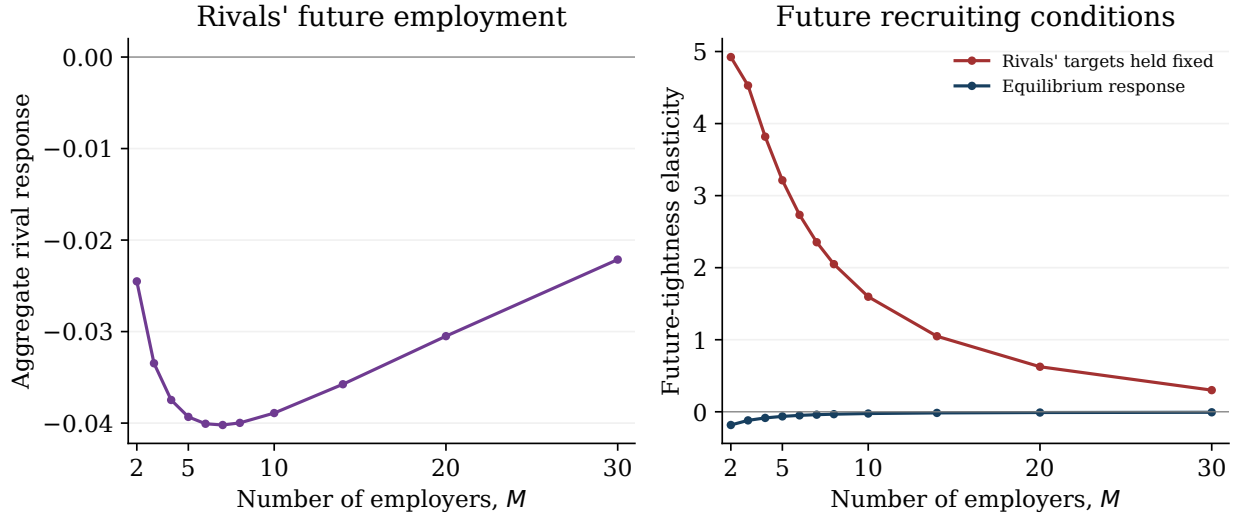


Figure 5: Rival hiring responses and future tightness

Notes: The left panel reports $\sum_{j \neq i} \partial n_{j,t+1} / \partial n_{it}$. The right panel reports $(n/\theta) \partial \theta_{t+1} / \partial n_{it}$. Holding rivals' targets fixed removes their local policy responses while retaining the own-policy and unemployment-pool responses.

Reduced rival hiring benefits the firm through recruiting costs but also changes its wage bill. In the recursive representation in (A.8), the recruiting benefit exceeds the wage effect under both duopoly and four employers (Table 5, panel B). This is why the strategic Euler contribution encourages hiring.

Employment shifts between firms persist longer than changes in aggregate employment. In the local equilibrium with two employers, an aggregate employment disturbance has a half-life of 10.5 weeks, whereas a redistribution of employment between the firms has a

half-life of 46.9 weeks. The corresponding half-lives with ten employers are 8.7 and 20.5 weeks. Rivals' contraction absorbs part of an initial employment increase at one firm, so the distributional effect persists after much of the aggregate effect has dissipated. Appendix C reports the transition paths implied by the full equilibrium policies.

5.5.2 Wage bargaining and future hiring exposure

Figure 6 separates the wage into its Stole–Zwiebel component and the additional general-equilibrium and strategic terms. The general-equilibrium wage contribution is slightly positive under monopoly and negative under oligopsony. At duopoly it is largely offset by positive strategic feedback. Both additional components vanish in the atomistic economy.

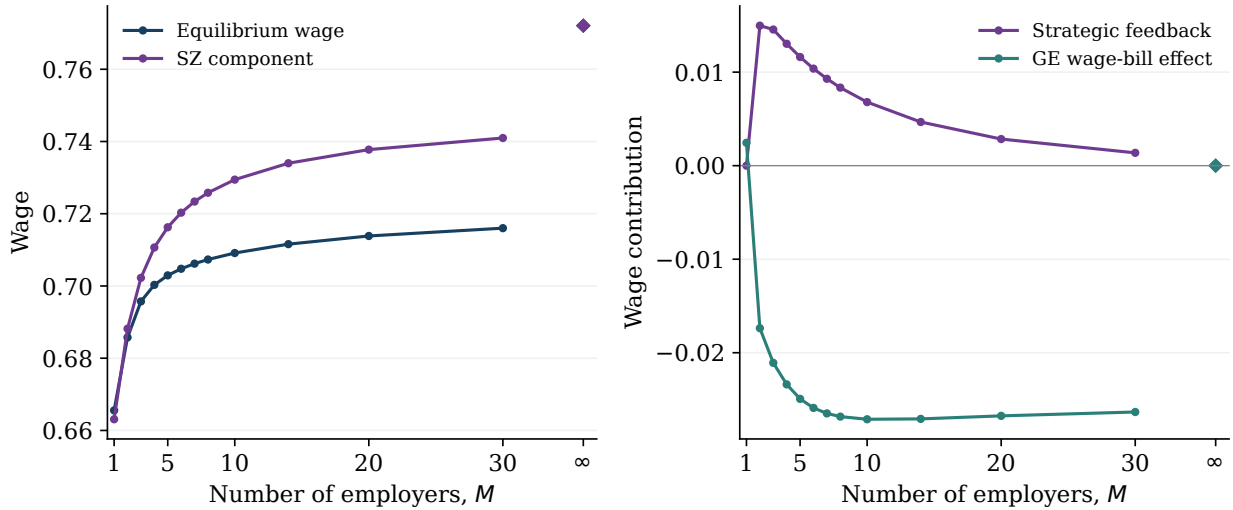


Figure 6: Wage determination and its additional equilibrium components

Notes: The left panel compares the wage with the atomistic Stole–Zwiebel component evaluated at the same finite-market allocation. The right panel shows the two remaining terms in (18). Their sum bridges the two lines in the left panel.

The relevant wage response for hiring is the own-employment derivative in (A.6). Figure 7 expresses each additional slope component in hiring Euler units, so their sum is the GE wage-bill effect in Figure 4. A negative contribution means that expansion saves less on the existing wage bill than under within-firm Stole–Zwiebel bargaining.

In the oligopsony equilibria, an increase in current employment reduces the firm's share of future hiring. The hiring-share term enters the wage equation with a negative coefficient, so this reduction raises the wage slope and discourages hiring. It is the largest contractionary component at both duopoly and four employers (Table 5, panel C). The future-tightness response partly offsets it.

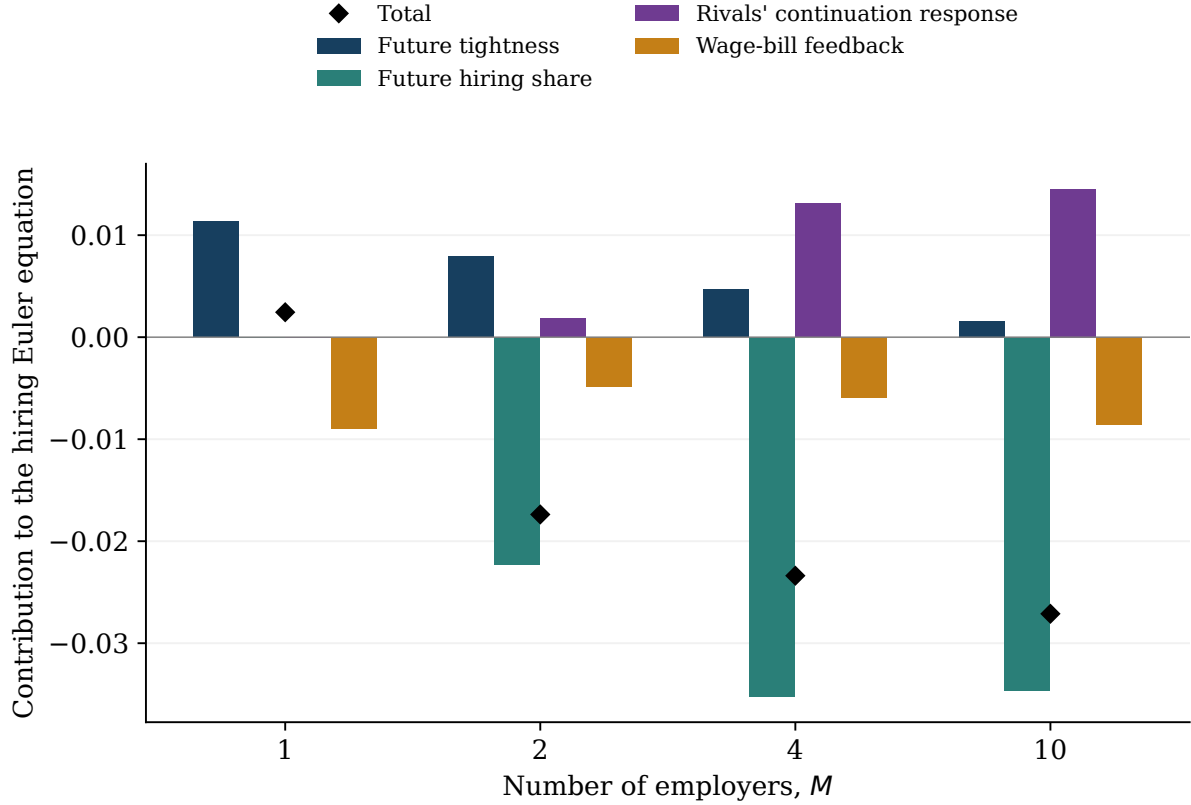


Figure 7: Components of the GE wage-bill effect on hiring

Notes: Bars are the four additional terms in (A.6), each multiplied by $-(1 - \chi)n$, where $n = N_M/M$. Diamonds give the GE wage-bill effect in (19). Positive values encourage hiring. The future-tightness component combines the worker and firm tightness terms; the future-hiring-share component isolates the change in the firm's recruiting exposure.

After including rivals' continuation responses and wage-bill feedback, the total GE wage-bill effect discourages hiring at both allocations. Wages still fall when the firm expands, but by less than under the canonical bargain.

The monopolist's future hiring share is always one, so its share derivative is zero. Future tightness and wage-bill feedback together produce a slightly positive GE wage-bill effect in Figure 7. Expansion saves slightly more on wages than under the canonical bargain, encouraging hiring. With a second employer, the firm's hiring share can respond, and the GE wage-bill effect changes sign. Farther along the curve, the combined responses of tightness, hiring shares, rivals, and the wage bill first strengthen and then weaken the contractionary effect. It vanishes with employer size along the local family in Proposition 1.

5.6 Employer size, concentration, and the fringe

I use the fringe to distinguish the effect of individual employer size from the effect of the fraction of workers employed by large firms. The comparisons retain the baseline parameters, zero UI, and the aggregate production frontier. Productive capacity is varied to match equilibrium hiring shares; wages, employment, and future policies adjust in each economy. Appendix G describes the computations.

The balance of recruiting and wage-bill incentives. Figure 8 compares the original economy with one large employer and an atomistic fringe on the same individual-size axis. At a 10 percent individual hiring share, the recruiting contribution changes from -0.0281 in the ten-employer economy to -0.0209 with a fringe, a 26 percent reduction in its contractionary effect. The GE wage-bill effect moves in the opposite direction, from -0.0271 to -0.0315 . Recruiting dominates at the more concentrated no-fringe allocations. With a fringe, the GE wage-bill effect is larger at the smaller individual hiring shares shown. Recruiting overtakes it between individual shares of 16.62 and 17.33 percent.

The recruiting contribution is already the small difference between current costs and replacement-hiring savings, together with the cost of a smaller future search pool. The bargaining offset proportional to $s(1 - S)$ in (35) reduces it further. The GE wage-bill effect depends on how current employment changes the firm's future hiring needs and continuation value, including the response of the fringe. It becomes more contractionary in the matched-share comparison.

The matched-size comparison also changes the fraction of hiring at large employers. Without a fringe, every hire is made by a large firm; with one large employer, the remaining

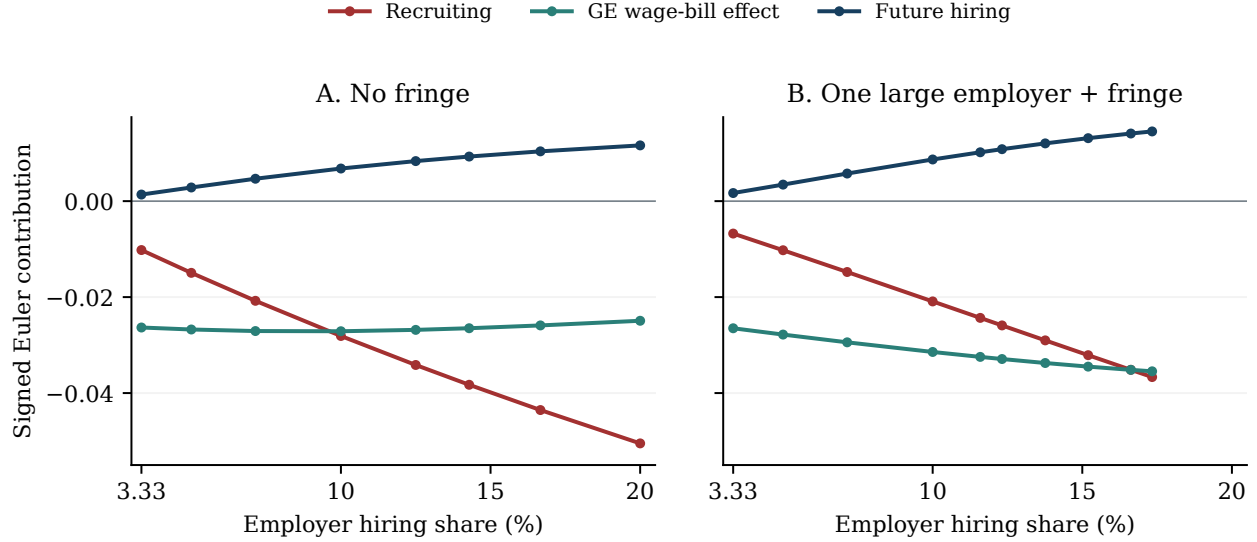


Figure 8: Hiring incentives with and without an atomistic fringe

Notes: The same calibration and efficient production frontier apply in both panels. The horizontal axis is an individual large employer’s stationary hiring share. Signed Euler contributions use the same sign and units as Figure 4: positive values encourage hiring. The recruiting line is T_i in (35). The future-hiring contribution comprises strategic large-rival responses without a fringe (left) and aggregate fringe adjustment with one large employer (right). Lines connect computed equilibria. The canonical multiworker component balances the three reported contributions.

hiring is atomistic. At the matched 10 percent individual share, unemployment is 6.327 percent without a fringe and 5.605 percent with one large employer and a fringe, compared with 5.517 percent in the atomistic economy. The worker-income gain from replacing large employers with atomistic employers falls from 8.99 to 1.09 percent.

Adding employers at a fixed individual size. Table 6 holds each large employer’s hiring share at 2.644 percent. Adding a second employer changes net recruiting by 1.8 percent and the GE wage-bill effect by less than 0.3 percent. Yet the unemployment gap relative to the atomistic benchmark approximately doubles, from 1.34 to 2.67 basis points. Each employer faces almost the same recruiting incentive, but twice as much hiring now takes place at large firms.

Equal concentration, different employer sizes. I also compare one large employer with two smaller employers at the same equilibrium HHI, choosing their capacities so that $s_1^2 = 2s_2^2$. At given matching conditions, (36) makes the hiring-weighted recruiting contribution proportional to HHI. After matching conditions adjust, the magnitude of the aggregate recruiting contribution differs by less than 0.21 percent across the three pairs. The aggregate GE wage-bill effect is instead 36–38 percent more contractionary with two employers. Their

Table 6: Adding a large employer at the same individual hiring share

	$M = 1$	$M = 2$
Individual hiring share (percent)	2.644	2.644
Combined large-employer share (percent)	2.644	5.287
Hiring HHI (10,000 $\mathcal{H}$)	6.989	13.978
Recruiting contribution	−0.005338	−0.005433
GE wage-bill effect	−0.02592	−0.02585
Future large-rival response	0	+0.000068
Future fringe response	+0.001010	+0.000938
Unemployment (percent)	5.531	5.544
Unemployment gap (basis points)	1.336	2.668

Notes: Productive capacity per large employer is 0.03219 with one large employer and 0.032 with two; capacity is chosen to match individual hiring shares. The four channel rows are T_i, G_i, R_i, F_i in (G.8), in the same signed Euler units as Figure 8. Large-rival feedback is shown separately from the fringe’s general-equilibrium response. The second employer replaces part of the fringe, increasing both HHI and the combined large-employer share. Unemployment in the atomistic benchmark is 5.517284 percent.

individual wage-bill incentives weaken only slightly, and together they account for $\sqrt{2}$ times as much hiring. At the largest matched HHI, unemployment is 5.538 percent with one large employer and 5.544 percent with two. The difference is 0.57 basis point, or 27 percent of the one-employer gap from the atomistic benchmark. Appendix G.6 reports all three pairs and their channel accounts.

Shrinking employers without changing their number. Figure 9 holds the number of large employers at one and reduces productive capacity. Both signed contributions approach zero from below as capacity shrinks. Dividing by capacity gives finite limiting values, with the GE wage-bill effect much larger in magnitude. As in Section 4.4, the disappearing contributions come from the firm’s influence on market conditions; ordinary within-firm Stole–Zwiebel wage dilution remains.

5.7 Efficient bargaining power

How much bargaining power should workers have if the aim is efficient employment? A large employer already pays for part of the congestion it creates, because a lower filling probability makes its own recruiting more expensive. Bargaining need only supply the private cost of the congestion imposed on other employers. Workers therefore need less bargaining power when their employer accounts for more of the market’s hiring.

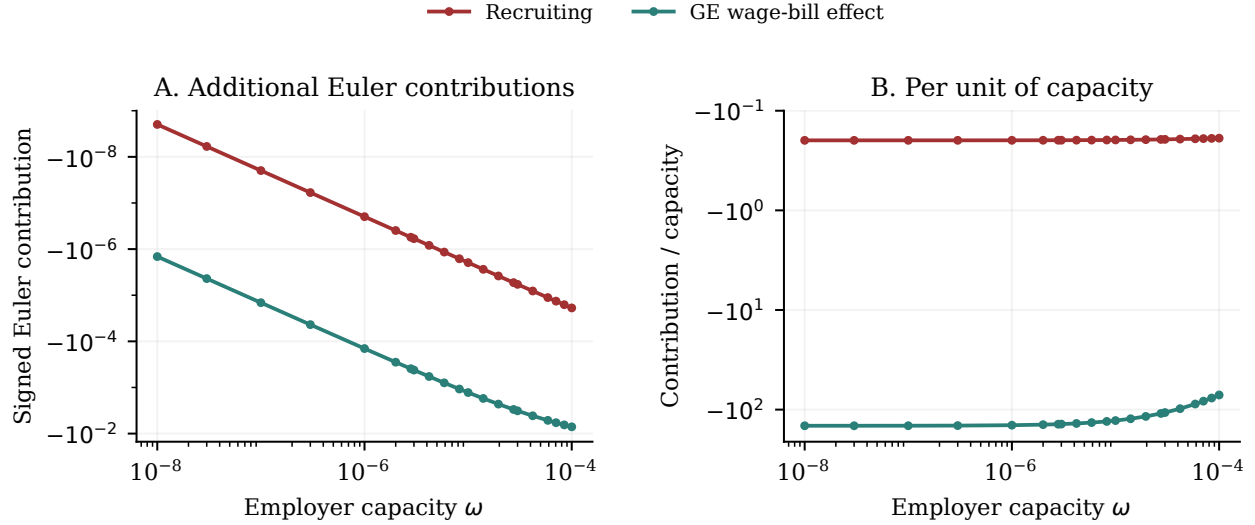


Figure 9: The small-employer limit with a competitive fringe

Notes: One large employer and an atomistic fringe. The left panel uses signed Euler units; the right divides these contributions by productive capacity. Negative values discourage hiring. Capacity and negative contribution magnitudes use logarithmic scales. Both contributions come from the regular bargaining solution and exclude canonical within-firm wage dilution.

Figure 10 quantifies this relationship. The matching-only condition, $\chi_M = \nu(1 - 1/M)$, isolates the correction for recruiting congestion. The complete condition also accounts for the wage-bill and strategic incentives derived above. At each market structure, I solve for the worker bargaining weight that implements the dynamic planner's employment target, allowing wages and continuation responses to adjust with bargaining power.

In the calibrated economy, the implementing worker bargaining weight rises from zero under monopoly to 0.334 with two employers, 0.505 with four, and 0.648 at the atomistic benchmark. Under monopoly, the employer bears all recruiting costs, and zero worker bargaining power makes its employment incentives coincide with those of the planner.

Away from monopoly, the complete condition in Figure 10 lies above the matching-only correction. The atomistic benchmark makes the reason particularly clear. Although an atomistic employer no longer affects market-wide recruiting conditions, expansion still lowers the wage paid to its existing workers through Stole–Zwiebel bargaining. This wage-bill saving encourages hiring. Correcting that incentive requires worker bargaining power above the matching elasticity: the implementing weight is 0.648, rather than $\nu = 0.5$.

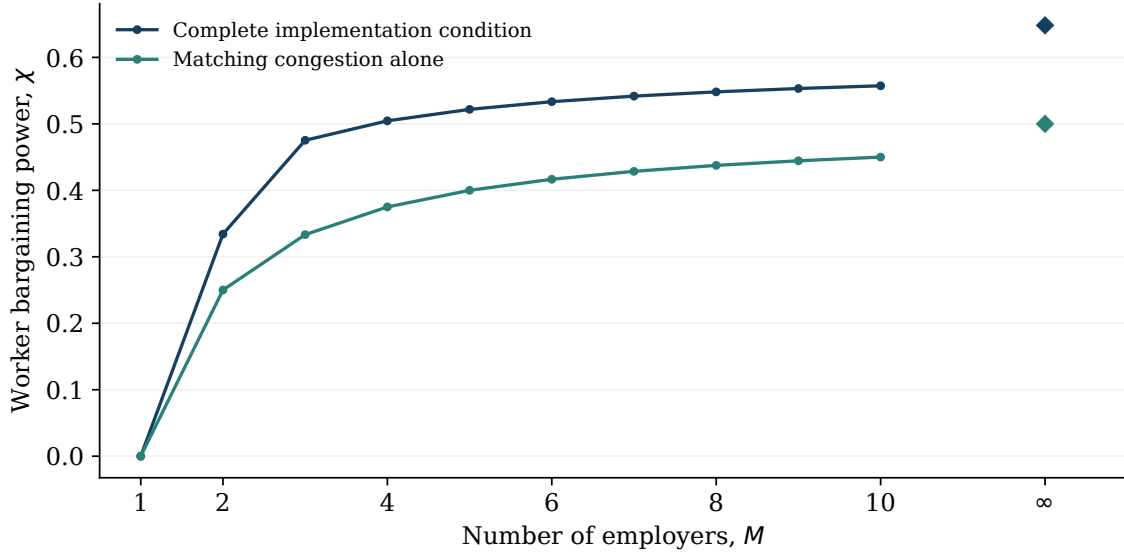


Figure 10: The Hosios condition and market structure

Notes: The matching-only line is $\nu(1 - 1/M)$. The complete line solves the planner-target employment condition jointly with the wage and continuation equations at each bargaining weight. The monopoly point is the analytic $\chi = 0$ boundary; the other points report $M = 2, \dots, 10$ and the atomistic benchmark.

5.8 Sensitivity to the calibration

Table 7 varies nonemployment value, production curvature, worker bargaining power, and the unemployment elasticity of matching. For each case, I refit matching efficiency and the recruiting cost to unemployment of 6 percent and a vacancy–unemployment ratio of 0.72 in the heterogeneous economy. I construct market-size weights jointly with that calibration and hold them fixed in its atomistic comparison. Each alternative therefore matches the same aggregate targets.

Across these calibrations, the weighted unemployment decline ranges from 0.41 to 0.56 percentage point. Aggregate welfare varies by 0.115 to 0.327 percent across market structures, and worker income rises by 11.7 to 20.4 percent from monopoly to the atomistic economy. Changing the parameters shifts the balance between recruiting and bargaining distortions. A higher b makes nonemployment more valuable, reducing the net gain from another job. A higher α weakens within-firm wage dilution and changes efficient employment.

5.9 A common policy across markets

In the calibrated economy, concentrated markets have too little employment and sufficiently unconcentrated markets have too much. A uniform increase in benefits reduces hiring in both by raising workers’ outside option. It corrects overemployment in the latter markets

Table 7: Employment, welfare, and incidence across calibrations

Case	Unemployment (%)		Weighted decline (pp)	Welfare range (%)	Worker-income gain (%)
	Monopoly	Atomistic			
Baseline	9.09	5.52	0.48	0.163	17.2
$b = 0.50$	8.95	5.46	0.54	0.115	11.7
$\alpha = 0.55$	8.94	5.44	0.56	0.138	15.0
$\alpha = 0.73$	9.24	5.59	0.41	0.327	19.1
$\chi = 0.40$	9.56	5.51	0.49	0.260	20.4
$\chi = 0.60$	8.80	5.52	0.48	0.261	14.9
$\nu = 0.40$	8.71	5.53	0.47	0.245	13.6
$\nu = 0.60$	9.19	5.53	0.47	0.272	20.2

Notes: The weighted decline replaces every market with its atomistic counterpart. The welfare range uses the same market types $M \in \{1, 2, 4, 10, 20, \infty\}$ in every case and is divided by that case’s monopoly aggregate welfare. The worker-income gain compares the atomistic and monopoly economies, relative to monopoly worker income excluding profits. Each row changes the indicated parameter and refits μ and κ .

but deepens underemployment in the former. The optimal common policy weighs these changes according to each market’s size, employment response, and social return to an additional job. Appendix C.7 compares common and market-specific policies. They can deliver approximately the same aggregate employment and still produce different welfare because the jobs are distributed differently across markets.

Under linear utility, these employment changes determine the welfare effect of UI. Risk-averse workers also value the insurance provided by benefits, and the wage response redistributes income between workers and profit recipients. I turn next to these gains and their interaction with hiring and search.

6 Optimal unemployment insurance in concentrated labor markets

How should unemployment insurance respond to employer concentration? Higher benefits protect unemployed workers against income loss, but workers also bargain and search differently when their nonemployment income rises. Firms respond by changing wages and hiring. I introduce risk aversion and endogenous search to study how these responses change the case for UI across market structures. The policy comparison holds the financing rule and social welfare weights fixed and varies the proportion of profits received by workers. This

determines whether wage gains come at the expense of workers' own profit income or transfer income from outside recipients.

6.1 Profit income, financing, and welfare

Workers have unit mass. A separate group of outside profit recipients has mass $1/9$, or 10 percent of the combined population. Workers receive the share $\phi \in [0, 1]$ of aggregate operating profits, distributed equally among employed and unemployed workers. Outside recipients receive the remaining share. Workers' profit income can come from ownership, an equal rebate of profit-tax revenue, or both. The parameter ϕ measures their total receipt from these sources. Appendix E.1 gives the income accounting and proportional profit-tax implementation.

The worker profit share is fixed within each policy exercise. I choose the increment b_0 to nonemployment income, financed by a common levy $G = b_0(1 - N)$ on every worker, employed or unemployed. This levy finances UI; profit transfers distribute firm income across the two population groups.

Positive b_0 provides insurance through a transfer to the unemployed. A negative value taxes nonemployment income and returns the proceeds through the common levy, which then becomes a rebate. Throughout the exercise, the underlying nonemployment value b remains fixed.

At a symmetric stationary allocation, operating profits net of wages and recruiting costs, and consumption after transfers, are

$$\pi = AN^\alpha - wN - \kappa V, \quad G = b_0(1 - N), \quad (44)$$

$$c_e = w + \phi\pi - G, \quad c_u = b + b_0 + \phi\pi - G, \quad (45)$$

$$c_K = 9(1 - \phi)\pi. \quad (46)$$

The unemployed pay the same levy as employed workers and receive the UI transfer. The UI budget balances in every period. Profit transfers exhaust operating profits, $\phi\pi + (1 - \phi)\pi = \pi$.

Workers and outside profit recipients have utility $u_\sigma(c)$. An unemployed worker supplies search effort e at cost $\Gamma(e)$, where

$$u_\sigma(c) = 1 + \frac{c^{1-\sigma} - 1}{1 - \sigma}, \quad \Gamma(e) = \frac{\gamma e^{1+\psi}}{1 + \psi}, \quad (47)$$

with the logarithmic extension at $\sigma = 1$. The normalization gives $u_0(c) = c$. The policy objective is stationary inclusive utilitarian welfare,

$$\mathcal{W} = Nu_\sigma(c_e) + (1 - N)[u_\sigma(c_u) - \Gamma(e)] + \frac{1}{9}u_\sigma(c_K). \quad (48)$$

Every person has the same welfare weight.²

With linear utility and fixed search, substituting (44)–(46) into (48) returns the aggregate resource criterion, up to any fixed search cost. With risk aversion, income distribution affects marginal utility and bargaining.

6.2 Search and nonlinear bargaining

Matching uses effective search:

$$H_t = \mu(e_t u_t)^\nu V_t^{1-\nu}, \quad q_t = \frac{H_t}{V_t}, \quad p_t = \frac{H_t}{u_t}. \quad (49)$$

Effort used in current matching is chosen in the preceding period and is predetermined when employers choose current targets. Current hiring changes next period's state and therefore future effort. An individual unemployed worker takes the contact rate per unit of effort and the destination distribution as given. For an interior choice,

$$\Gamma'(e_{t+1}) = \frac{p_{t+1}}{e_{t+1}} \sum_j \frac{h_{j,t+1}}{H_{t+1}} S_{j,t+1}. \quad (50)$$

The surplus is now measured in utility units. Bilateral bargaining becomes

$$(1 - \chi)S_{it} = \chi u'_\sigma(c_{e,it})J_{it}. \quad (51)$$

An atomistic worker treats the common profit transfer and levy as fixed in the bilateral negotiation. Both vary with the aggregate state and enter the equilibrium derivatives. The term $u'_\sigma(c_{e,it})$ converts the firm's monetary surplus into worker utility. Appendix E gives the worker recursion and the corresponding firm and effort conditions.

Workers who receive more profit income have different consumption and marginal utility, which changes their bargaining and search decisions. The profit share therefore affects wages, hiring, and effort as well as the welfare value of their responses to UI.

²When workers receive all profits ($\phi = 1$), the outside group receives no income in this specification. Under the calibrated $\sigma = 0.5$, its contribution to welfare is the finite constant $u_\sigma(0)/9 = -1/9$. This constant does not affect comparisons across UI policies at $\phi = 1$.

6.3 The welfare effect of UI

An increase in UI transfers income to unemployed workers and changes the equilibrium wage, employment, and search effort. Differentiating stationary welfare separates these effects. Write $N_b = dN/db_0$, $w_b = dw/db_0$, and $e_b = de/db_0$. Define the average marginal value of worker income and the social marginal value of profits as

$$a = Nu'_\sigma(c_e) + (1 - N)u'_\sigma(c_u), \quad \lambda = \phi a + (1 - \phi)u'_\sigma(c_K). \quad (52)$$

When all profits reach workers, $\lambda = a$. Let π_N and π_e denote partial derivatives of profits holding w and, respectively, e or N fixed. The exact derivative is

$$\begin{aligned} \frac{dW}{db_0} = & \underbrace{(1 - N)[u'_\sigma(c_u) - a]}_{\text{direct insurance}} \\ & + \underbrace{[u_\sigma(c_e) - u_\sigma(c_u) + \Gamma(e) + \lambda\pi_N + b_0a]N_b}_{\text{employment and output}} \\ & + \underbrace{N[u'_\sigma(c_e) - \lambda]w_b}_{\text{wage and rent redistribution}} \\ & + \underbrace{[\lambda\pi_e - (1 - N)\Gamma'(e)]e_b}_{\text{search and recruiting}}. \end{aligned} \quad (53)$$

The first term is the direct insurance gain net of financing. It equals $N(1 - N)[u'_\sigma(c_u) - u'_\sigma(c_e)]$ and is positive when unemployed workers consume less than employed workers.

In the employment-and-output contribution, the term in brackets measures the welfare value of an additional employed worker, holding the wage and search effort fixed. It includes the change in the worker's consumption and search costs, the additional profits generated by employment, and the change in UI financing. Multiplying this value by N_b gives the welfare effect of the employment response to UI. A large employment cost can reflect a strong reduction in hiring, a high social value of the jobs lost, or both. The numerical analysis measures these two factors separately.

The third term measures wage and rent redistribution. A higher wage benefits employed workers but reduces profits. Its welfare value depends on the difference between the marginal utility of employed workers and the marginal social value of income received as profits, $u'_\sigma(c_e) - \lambda$, as well as on the wage response w_b .

When workers receive all profits, employed workers receive the wage increase, but both employed and unemployed workers lose profit transfers. When unemployed workers have

higher marginal utility, this redistribution reduces welfare. When workers receive no profits, higher wages also transfer income from outside profit recipients to employed workers. This transfer raises welfare when employed workers value the additional income more than its recipients would have valued it as profits. The fourth term combines the disutility of search with its effect on recruiting resources.

As in Landais, Michailat, and Saez (2018), insurance must be weighed against behavioral responses and their general-equilibrium effects. Lower search effort saves effort costs but also changes employment and UI financing. Those consequences enter the search and employment contributions, respectively. Wage redistribution adds a gain or loss according to who receives profits.

I evaluate (53) in the equilibrium for each profit-income arrangement. Appendix E derives the expression and the equilibrium derivatives.

6.4 Optimal UI across market structures

The risk-averse economy retains the baseline production and matching elasticities, bargaining power, discount factor, and separation rate. I use $\sigma = 0.5$ as a moderate risk-aversion benchmark and set the search-cost curvature to $\psi = 0.6$. The latter corresponds to the estimated curvature of search cost in job-finding probability in the standard model of DellaVigna et al. (2022), Table VI. Individual effective search effort scales job-finding probability linearly in the present model, so this is the relevant empirical counterpart. Appendix E.8 varies both preference parameters.

Recruiting cost, matching efficiency, and the search-cost scale are fitted at $\phi = 1$ under common-worker financing, at $b_0 = 0$. The targets are unemployment of 6 percent, a vacancy–unemployment ratio of 0.72, and mean search effort among unemployed workers of one. The last condition normalizes mean effective search, allowing effort to vary across markets. This gives $\kappa = 0.7709$, $\mu = 0.08887$, and $\gamma = 0.5995$. Risk aversion and search curvature are held fixed in fitting these moments. The calibration uses the complete degree-one heterogeneous system; policy exercises use degree three with the same primitives. Appendix E.7 reports the aggregate fit at both approximation orders.

Figure 11 reports the policy maximizing (48) on each admissible equilibrium branch. The finite-market exercise uses degree-three equilibria for $M = 1, \dots, 10$ and an exact atomistic benchmark. Parameters and the financing rule are held fixed across profit-income arrangements.

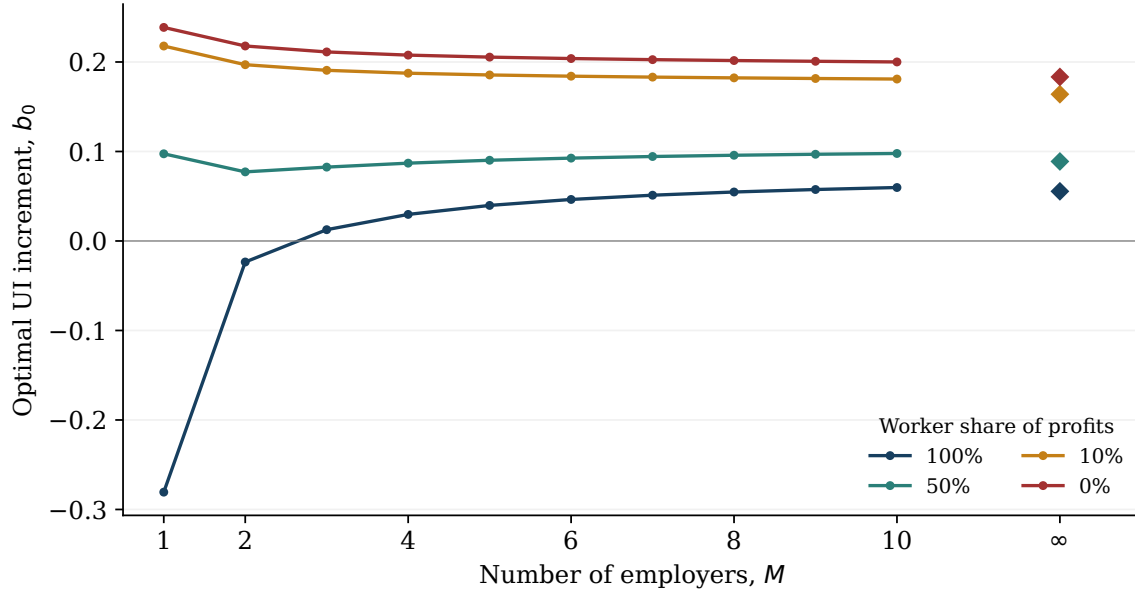


Figure 11: Optimal UI and the distribution of profit income

Notes: The objective is (48), with $\sigma = 0.5$, $\psi = 0.6$, common-worker financing, and outside-recipient mass $1/9$. Lines join the finite-market solutions. Diamonds denote the atomistic benchmark. The instrument b_0 is an increment relative to $b = 0.4$. All optima are interior to the searched UI interval, with positive worker consumption and negative local objective curvature.

Table 8: Optimal UI, consumption, and unemployment

Worker profit share	M	b_0	Unemployment (%)	c_e	c_u	Levy G
100%	1	-0.281	7.71	0.978	0.704	-0.022
100%	∞	+0.055	6.99	0.965	0.761	+0.004
50%	1	+0.097	16.75	0.780	0.660	+0.016
50%	∞	+0.089	7.89	0.809	0.637	+0.007
10%	1	+0.218	25.31	0.671	0.592	+0.055
10%	∞	+0.164	11.64	0.686	0.575	+0.019
0%	1	+0.239	27.40	0.646	0.573	+0.065
0%	∞	+0.183	13.14	0.657	0.559	+0.024

Notes: Each row maximizes stationary inclusive welfare under common-worker financing. Consumption includes profit income and the levy. Negative b_0 taxes nonemployment income; a negative levy rebates the proceeds to all workers.

When workers receive all profits, optimal benefits are lower in more concentrated markets. The monopoly optimum involves a negative increment, whereas the atomistic optimum involves a positive one. In this calibration, reducing the worker profit share raises optimal UI. When all profits go to outside recipients, the ranking reverses and the monopoly optimum provides more generous benefits than the atomistic optimum. At a 50 percent worker share, optimal benefits fall from monopoly to duopoly and then rise across the remaining finite-market points.

When workers receive all profits, higher UI reduces the income available to them by lowering employment and output, as in the linear economy. This cost favors lower benefits in concentrated markets, where employment is initially too low. Higher wages also reduce the profit income shared with unemployed workers. That income already provides some insurance, reducing the need for UI.

When workers receive little profit income, higher wages transfer income from richer profit recipients to workers with higher marginal utility. Wages respond more strongly to UI in concentrated markets, making this redistribution gain larger there. With no profits reaching workers, moving from zero UI to the monopoly optimum raises unemployment from 12.62 to 27.40 percent (Table 8). At the optimum, consumption is 0.646 for employed workers and 0.573 for unemployed workers, about 89 percent as much. Profit recipients are a small, richer group with lower marginal utility than workers, so under the calibrated $\sigma = 0.5$, shifting income from them to workers through higher negotiated wages has high welfare value. The common worker levy finances the benefits themselves, which bring unemployed workers' consumption close to that of employed workers.

The monopoly–atomistic policy ordering persists when risk aversion, search-cost curvature, and outside-recipient income vary (Appendix E.8).

6.5 Why the policy gradient changes with profit income

Figure 12 compares the welfare contributions at the same policy, $b_0 = 0$, across market structures. Equilibrium wages, employment, search, and marginal utilities adjust to each profit-income arrangement.

With all profits going to workers, employment accounts for much of the difference across market structures. Starting from the common zero policy, a benefit increase destroys more jobs under monopoly, and each lost job has a higher welfare value. In the atomistic economy, employment also falls, but this raises welfare at the margin when wages and search are

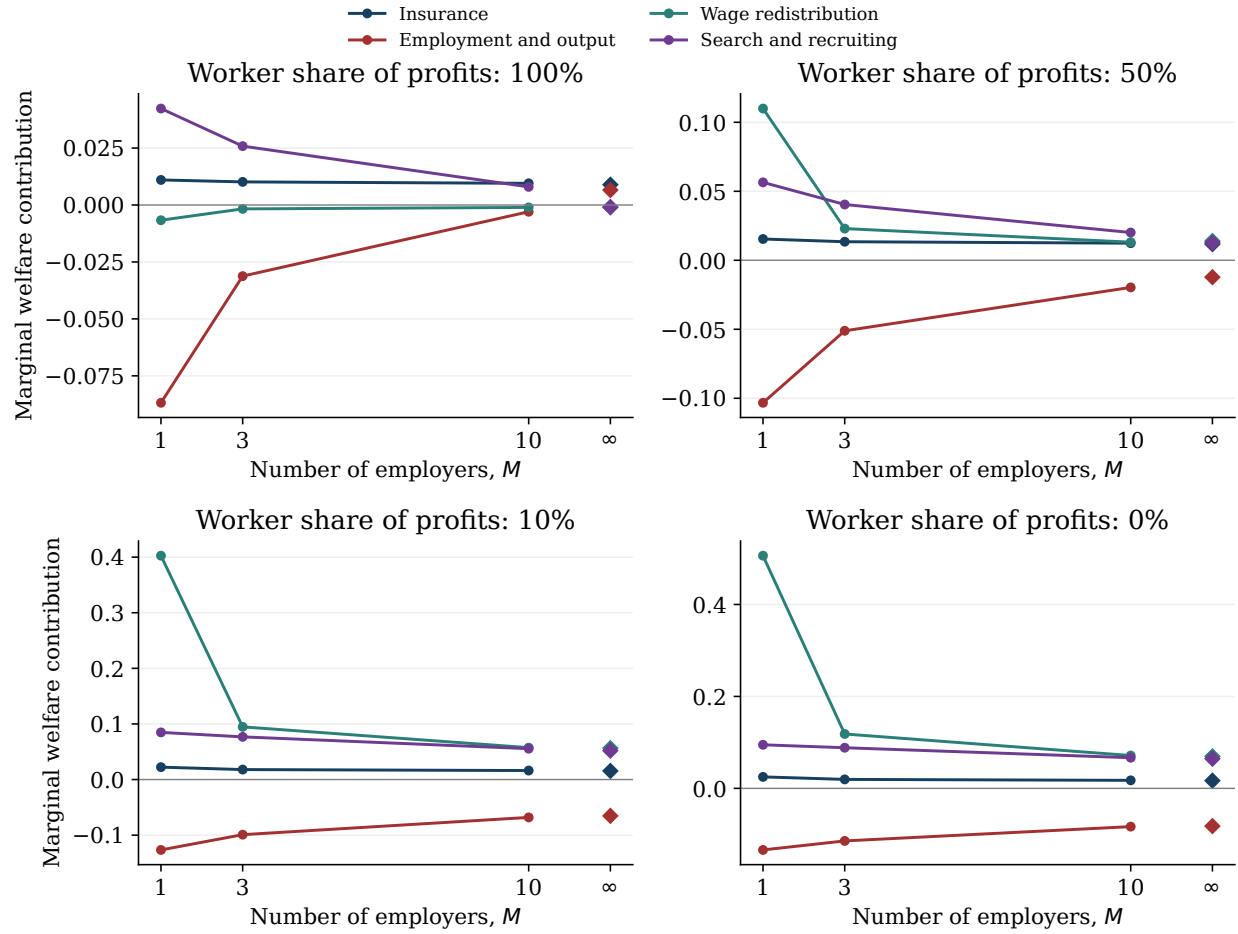


Figure 12: UI welfare channels across market structures

Notes: Contributions to (53) at $b_0 = 0$. Vertical scales differ across panels. Appendix E reports the numerical contributions and derivative checks.

held fixed. Its employment contribution is therefore positive. Appendix E.10 separates the employment response from the welfare value multiplying it.

Why is another job worth more under monopoly? With lower employment, its marginal product is higher, and firms can recruit from a larger unemployment pool. The larger pool accounts for most of the difference in the net resources generated by an additional job. With all profits reaching workers, the marginal recruiting expenditure needed to sustain additional stationary employment, κV_N , is 0.198 under monopoly and 0.678 in the atomistic economy at $b_0 = 0$, holding search effort fixed. Filling vacancies is harder in the atomistic economy's smaller search pool, so sustaining another job uses more recruiting resources. The worker's consumption gain and the search costs avoided by taking a job also enter its welfare value.

When workers receive no profits, the welfare value of an additional job is nearly the same under monopoly and in the atomistic economy. The larger employment cost under monopoly comes almost entirely from its stronger hiring response. Wage redistribution nevertheless dominates the concentration comparison. At $\phi = 0$ and $b_0 = 0$, the wage response per unit increase in UI, w_b , is 0.688 under monopoly and 0.111 in the atomistic economy. The welfare value of shifting a dollar from profits to wages differs much less across the two markets. Most of the larger wage-redistribution contribution under monopoly comes from the stronger wage response.

Higher UI narrows the consumption gap between employment and unemployment, putting upward pressure on negotiated wages. Workers also search less. With lower effort, a smaller utility advantage from employment suffices to satisfy their search condition, offsetting some of that wage pressure. Table 9 reports the two forces for these same equilibria. Although the direct upward pressure is larger in the atomistic economy, search adjustment offsets much more of it, leaving a smaller net wage response. Vacancy-filling adjustments account for the remainder. Appendix E.12 derives the components.

Table 9: Accounting for the wage response to UI

Contribution	Monopoly	Atomistic
Direct UI effect	+1.1017	+1.2167
Search adjustment	−0.3920	−1.0703
Vacancy-filling adjustment	−0.0220	−0.0352
Total wage response, w_b	+0.6877	+0.1112

Notes: Signed contributions to the wage response per unit increase in UI, evaluated at $\phi = 0$ and $b_0 = 0$. The three components sum to w_b ; the separate profit-transfer and financing terms in (E.18) are zero at this allocation.

Intermediate profit shares combine these patterns. At a 50 percent worker share, the changes in wage redistribution and employment nearly offset, leaving insurance and search to determine the remaining difference. The corresponding optimal-policy schedule is non-monotone across the finite-market points in Figure 11.

The comparison at zero UI explains the initial direction of policy. Employment responses change as benefits approach their optimum. Under monopoly with full worker profit income, successive benefit cuts bring smaller employment gains. With no worker profit income, successive increases bring larger employment losses. At the optimum, the marginal employment effect balances insurance, redistribution, and search. Appendix E.11 traces the contributions along each policy path and accounts for the resulting welfare gain.

7 Conclusion

Hiring today changes the labor market in which an employer will recruit tomorrow. In this DMP economy, firms anticipate both the cost of filling their own vacancies and their competitors' responses to tomorrow's employment state. Workers bargain over the resulting surplus, so these hiring decisions also change wages across the existing workforce.

Correcting matching congestion requires less worker bargaining power when employers already bear much of its cost on their own recruiting. The complete Hosios condition also accounts for wage-bill and strategic incentives. The local convergence result connects this economy to the atomistic multiworker benchmark as employers become small. Firms then lose their individual influence on market conditions but continue to gain from within-firm wage dilution.

In the calibrated market distribution, replacing large employers with atomistic ones lowers unemployment by about 0.48 percentage point. The decline ranges from 0.41 to 0.56 percentage point across alternative calibrations. Worker income excluding profits rises by 4.34 percent, yet aggregate welfare falls by 0.03 percent. Many markets begin near efficient employment, and atomistic employers expand beyond it to reduce wages within the firm. These overhiring losses outweigh the gains in more concentrated markets. Workers benefit chiefly from the redistribution of profits through higher wages.

With a competitive fringe, individual employer size can vary separately from the share of hiring controlled by large firms. HHI summarizes the hiring-weighted recruiting contribution at given matching conditions. Wage-bill incentives and future hiring responses also depend on the distribution of employment. In the equal-HHI exercise, unemployment differs by up to 0.57 basis point, or 27 percent of the gap between the market with one large employer and

the atomistic benchmark. For merger analysis, the comparison suggests examining which employers gain hiring share, alongside the change in HHI.

The UI results turn on how workers receive the economy's income. When all profits reach workers through ownership or redistribution, a benefit increase costs them jobs, and the higher wage reduces profit income shared with the unemployed. Concentrated markets then favor lower benefits. When profits go to outside recipients, higher wages transfer income to workers. Because wages respond more strongly to UI in concentrated markets, those markets favor more generous UI in the calibration. As benefits rise, growing employment losses eventually balance the net gains from insurance, redistribution, and reduced search.

Evidence on job finding and firm-level recruiting responses would help quantify how employers' influence on the unemployment pool affects hiring.

References

- Acemoglu, Daron, and William B. Hawkins. 2014. “Search with Multi-Worker Firms.” *Theoretical Economics* 9(3): 583–628.
- Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen. 2020. “The Fall of the Labor Share and the Rise of Superstar Firms.” *Quarterly Journal of Economics* 135(2): 645–709.
- Bergeaud, Antonin, Pierre Cahuc, Clément Malgouyres, and Sara Signorelli. “Oligopsony in Frictional Labor Markets.” Work in progress, listed on the authors’ research pages. <https://sites.google.com/view/sarasignorelli/research>.
- Berger, David, Kyle Herkenhoff, and Simon Mongey. 2022. “Labor Market Power.” *American Economic Review* 112(4): 1147–1193. <https://doi.org/10.1257/aer.20191521>.
- Berger, David, Kyle Herkenhoff, and Simon Mongey. 2026. “Labor Market Power: From Micro Evidence to Macro Consequences.” *Journal of Economic Perspectives* 40(1): 93–114. <https://doi.org/10.1257/jep.20251456>.
- Berger, David, Kyle Herkenhoff, Andreas R. Kostøl, and Simon Mongey. 2024. “An Anatomy of Monopsony: Search Frictions, Amenities, and Bargaining in Concentrated Markets.” *NBER Macroeconomics Annual* 38. <https://doi.org/10.1086/729194>.
- Bertolotti, Fabio, Andrea Lanteri, and Alessandro Villa. 2024. “Investment-Goods Market Power and Capital Accumulation.” Federal Reserve Bank of Chicago Working Paper 2024-13, revised July 2026. <https://www.chicagofed.org/publications/working-papers/2024/2024-13>.
- Bloesch, Justin, Birthe Larsen, and Anders Yding. 2026. “Monopsony with Recruiting.” Working paper, revised March 2026. <https://ssrn.com/abstract=5020154>.
- De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. 2020. “The Rise of Market Power and the Macroeconomic Implications.” *Quarterly Journal of Economics* 135(2): 561–644. <https://doi.org/10.1093/qje/qjz041>.
- DellaVigna, Stefano, Jörg Heining, Johannes F. Schmieder, and Simon Trenkle. 2022. “Evidence on Job Search Models from a Survey of Unemployed Workers in Germany.” *Quarterly Journal of Economics* 137(2): 1181–1232. <https://doi.org/10.1093/qje/qjab039>.
- Elsby, Michael W. L., and Ryan Michaels. 2013. “Marginal Jobs, Heterogeneous Firms, and Unemployment Flows.” *American Economic Journal: Macroeconomics* 5(1): 1–48. <https://doi.org/10.1257/mac.5.1.1>.
- Gottfries, Axel, and Gregor Jarosch. 2026. “Dynamic Monopsony with Granular Firms.” NBER Working Paper 31965, revised April 2026. <https://www.nber.org/papers/w31965>. Forthcoming as “Dynamic Monopsony with Large Firms,” *Econometrica*.

- Hosios, Arthur J. 1990. “On the Efficiency of Matching and Related Models of Search and Unemployment.” *Review of Economic Studies* 57(2): 279–298.
- Hurst, Erik, Patrick J. Kehoe, Elena Pastorino, and Thomas Winberry. 2025. “The Macroeconomic Dynamics of Labor Market Policies.” NBER Working Paper 33614. <https://www.nber.org/papers/w33614>.
- Jarosch, Gregor, Jan Sebastian Nimczik, and Isaac Sorkin. 2024. “Granular Search, Market Structure, and Wages.” *Review of Economic Studies* 91(6): 3569–3607.
- Klein, Paul, Per Krusell, and José-Víctor Ríos-Rull. 2008. “Time-Consistent Public Policy.” *Review of Economic Studies* 75(3): 789–808.
- Landais, Camille, Pascal Michaillat, and Emmanuel Saez. 2018. “A Macroeconomic Approach to Optimal Unemployment Insurance: Theory.” *American Economic Journal: Economic Policy* 10(2): 152–181. <https://doi.org/10.1257/pol.20150088>.
- Manning, Alan. 2004. “Monopsony and the Efficiency of Labour Market Interventions.” *Labour Economics* 11(2): 145–163.
- Mortensen, Dale T., and Christopher A. Pissarides. 1994. “Job Creation and Job Destruction in the Theory of Unemployment.” *Review of Economic Studies* 61(3): 397–415.
- Naidu, Suresh, and Eric A. Posner. 2022. “Labor Monopsony and the Limits of the Law.” *Journal of Human Resources* 57(S): S284–S323. <https://jhr.uwpress.org/content/57/S/S284>.
- Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory*. Second edition. Cambridge, MA: MIT Press.
- Shimer, Robert. 2005. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies.” *American Economic Review* 95(1): 25–49.
- Stole, Lars A., and Jeffrey Zwiebel. 1996. “Intra-Firm Bargaining under Non-Binding Contracts.” *Review of Economic Studies* 63(3): 375–410. <https://doi.org/10.2307/2297888>.

A Equilibrium and efficiency derivations

This appendix derives the equations used in the main text. Partial derivatives refer to labeled employment coordinates before symmetry is imposed. A current hiring deviation holds rivals' current targets fixed. A derivative with respect to the inherited state instead differentiates their Markov policies.

A.1 Recruiting derivatives and the envelope

Write $\mathbf{k} = \mathbf{n}_{t-1}$ and $\mathbf{n} = \mathcal{N}(\mathbf{k})$. Then

$$u = 1 - \sum_j k_j, \quad h_i = n_i - (1 - \delta)k_i, \quad H = \sum_j h_j, \quad q^{-1} = \mu^{-1} \left(\frac{H}{\mu u} \right)^{\nu/(1-\nu)}.$$

At fixed $\mathbf{k}$ and fixed rival targets,

$$\frac{\partial q^{-1}}{\partial n_i} = q^{-1} \frac{\nu}{1 - \nu} \frac{1}{H}, \quad \frac{\partial(h_i/q)}{\partial n_i} = q^{-1} \left(1 + \frac{\nu}{1 - \nu} \frac{h_i}{H} \right).$$

Differentiating profits gives (14). Holding all current targets fixed while differentiating k_k gives

$$\frac{\partial \log q^{-1}}{\partial k_k} = \frac{\nu}{1 - \nu} \left(\frac{1}{u} - \frac{1 - \delta}{H} \right), \quad \frac{\partial h_i}{\partial k_k} = -(1 - \delta) \mathbf{1}\{i = k\}.$$

The chain-rule derivative through firm i 's target is zero at an interior optimum. The derivative through each rival j remains and equals

$$\frac{\partial \mathcal{N}_j(\mathbf{k})}{\partial k_k} \left[\beta \frac{\partial \Pi_i(\mathbf{n})}{\partial n_j} - n_i \frac{\partial w_i(\mathbf{n})}{\partial n_j} - h_i \frac{\nu}{1 - \nu} \frac{\kappa}{qH} \right].$$

Combining direct and induced effects gives (15). The own-state derivative contains the replacement-hiring term. The cross-state derivative does not, but it retains the same aggregate-pool and rival-policy effects.

A.2 The wage equation away from symmetry

Subtracting unemployed value from employed value yields

$$S_{it} = w_{it} - b - b_0 + \beta \left[(1 - \delta) S_{i,t+1} - p_{t+1} \sum_j \frac{h_{j,t+1}}{H_{t+1}} S_{j,t+1} \right]. \quad (\text{A.1})$$

The first-order condition implies

$$J_{it} = \frac{\kappa}{q_t} \left(1 + \frac{\nu}{1-\nu} \frac{h_{it}}{H_t} \right). \quad (\text{A.2})$$

Substitute the bargaining rule into (A.1) and use (A.2) at future dates. This gives the alternative firm row

$$\begin{aligned} 0 = & \frac{1-\chi}{\chi} (w_{it} - b - b_0) - \frac{\kappa}{q_t} \left(1 + \frac{\nu}{1-\nu} \frac{h_{it}}{H_t} \right) \\ & + \beta(1-\delta) \frac{\kappa}{q_{t+1}} \left(1 + \frac{\nu}{1-\nu} \frac{h_{i,t+1}}{H_{t+1}} \right) \\ & - \beta\kappa\theta_{t+1} \left[1 + \frac{\nu}{1-\nu} \sum_j \left(\frac{h_{j,t+1}}{H_{t+1}} \right)^2 \right]. \end{aligned} \quad (\text{A.3})$$

It holds for $0 < \chi < 1$ and is equivalent to the raw firm condition when the bargaining and envelope equations hold. At the monopoly boundary $\chi = 0$, I use the raw equations directly.

Using the own envelope to eliminate the own continuation value gives the full wage equation

$$\begin{aligned} w_{it} = & (1-\chi)(b + b_0) + \chi \left[\alpha A_i n_{it}^{\alpha-1} - n_{it} \frac{\partial w_i(\mathbf{n}_t)}{\partial n_{it}} \right] \\ & + \chi\beta\kappa\theta_{t+1} \left[1 + \frac{\nu}{1-\nu} \sum_j \left(\frac{h_{j,t+1}}{H_{t+1}} \right)^2 \right] - \chi\beta\kappa \frac{\nu}{1-\nu} \theta_{t+1} \frac{h_{i,t+1}}{H_{t+1}} \\ & + \chi\beta \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}} \left[\beta \frac{\partial \Pi_i(\mathbf{n}_{t+1})}{\partial n_{j,t+1}} - n_{i,t+1} \frac{\partial w_i(\mathbf{n}_{t+1})}{\partial n_{j,t+1}} - h_{i,t+1} \frac{\nu}{1-\nu} \frac{\kappa}{q_{t+1} H_{t+1}} \right]. \end{aligned} \quad (\text{A.4})$$

At symmetry every hiring share is $1/M$. The sum of squared shares and the firm's own share have the same level. Their level terms in the second line then reduce to $\chi\beta\kappa\theta$. Add and subtract the canonical slope to obtain (18).

For derivatives, however,

$$\left. \frac{\partial}{\partial n_{it}} \sum_j \left(\frac{h_{j,t+1}}{H_{t+1}} \right)^2 \right|_{\text{sym}} = \frac{2}{M} \frac{\partial}{\partial n_{it}} \sum_j \frac{h_{j,t+1}}{H_{t+1}} = 0,$$

whereas the own-share derivative can be nonzero. Differentiating (A.4), subtracting the canonical wage differential equation, and then evaluating at symmetry gives the unilateral wage derivative in (A.6).

A.3 The regular Stole–Zwiebel solution

Holding the aggregate continuation environment fixed, the production part of the wage equation is

$$w^{\text{prod}}(x) + \chi x \frac{dw^{\text{prod}}(x)}{dx} = \chi \alpha A_i x^{\alpha-1}.$$

Its regular particular solution is

$$w^{\text{prod}}(x) = \frac{\chi \alpha A_i}{1 - \chi(1 - \alpha)} x^{\alpha-1}.$$

The homogeneous term is $Cx^{-1/\chi}$. Because $0 < \chi < 1$, its payroll $Cx^{1-1/\chi}$ diverges at zero unless $C = 0$. The regular production payroll instead tends to zero. The maintained convention selects $C = 0$, as in the standard multiworker solution.

For a bounded nonproduction remainder $f(x)$, the equation $v + \chi x v_x = f$ has the regular inverse

$$(\mathcal{I}_\chi f)(x) = \frac{1}{\chi} \int_0^1 t^{1/\chi-1} f(tx) dt, \quad \|\mathcal{I}_\chi f\|_\infty \leq \|f\|_\infty. \quad (\text{A.5})$$

The integral requires the proof's marked domain to include the complete own-employment wage segment from zero to the target.

A.4 The wage response to own employment

The full own-employment wage slope is obtained by differentiating the wage equation *before* imposing symmetry:

$$\begin{aligned} \left. \frac{\partial w_i}{\partial n_{it}} \right|_{\text{sym}} &= \left. \frac{\partial w_i}{\partial n_i} \right|_{\text{SZ}} + \beta \chi \kappa \left. \frac{\partial \theta_{t+1}}{\partial n_{it}} \right|_{\text{sym}} \\ &\quad - \beta \chi \kappa \frac{\nu}{1 - \nu} \theta_{t+1} \left. \frac{\partial}{\partial n_{it}} \left(\frac{h_{i,t+1}}{H_{t+1}} \right) \right|_{\text{sym}} \\ &\quad + \chi \beta \frac{\partial}{\partial n_{it}} \left\{ \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}} \left[\beta \frac{\partial \Pi_i(\mathbf{n}_{t+1})}{\partial n_{j,t+1}} - n_{i,t+1} \frac{\partial w_i(\mathbf{n}_{t+1})}{\partial n_{j,t+1}} \right. \right. \\ &\quad \quad \left. \left. - h_{i,t+1} \frac{\nu}{1 - \nu} \frac{\kappa}{q_{t+1} H_{t+1}} \right] \right\} \Big|_{\text{sym}} \\ &\quad - \chi \frac{\partial}{\partial n_{it}} \left\{ n_{it} \left[\frac{\partial w_i(\mathbf{n}_t)}{\partial n_{it}} - \left. \frac{\partial w_i}{\partial n_i} \right|_{\text{SZ}} \right] \right\} \Big|_{\text{sym}}. \end{aligned} \quad (\text{A.6})$$

The terms are, respectively, the canonical slope, the future-tightness response, the firm's future recruiting exposure, the change in rivals' continuation responses, and inframarginal

wage-bill feedback. More employment today changes workers' outside options by affecting future tightness. It also changes how much future recruiting the firm undertakes. Under monopoly, the hiring share is identically one, leaving future tightness and wage-bill feedback as the additional wage effects. A second employer makes the hiring share responsive to employment, which can produce a discrete change between monopoly and duopoly.

The wage-bill feedback in (A.6) contains the second derivative of the wage schedule. The numerical method solves the wage schedule and its derivatives jointly. The convergence proof bounds these responses near the atomistic economy.

A.5 The stationary Euler equation and strategic recursion

Combining (14) with the own envelope one period forward gives the current and future recruiting terms. Substitute (18) and collect the wage-slope terms:

$$-n \frac{\partial w_i}{\partial n_i} + \chi n \frac{\partial w_i}{\partial n_i} = -(1 - \chi) n \frac{\partial w_i}{\partial n_i}.$$

At symmetry $h_i/H = 1/M$ and $h_i/u = p/M$. The resulting recruiting correction is

$$-\frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} + \frac{\beta}{M} (1 - \delta) \frac{\nu}{1 - \nu} \frac{\kappa}{q} - \frac{\beta}{M} p \frac{\nu}{1 - \nu} \frac{\kappa}{q},$$

which is (20). The strategic wage term absorbs the fraction χ of the firm strategic term, leaving the fraction $1 - \chi$ in (19).

Define two sums of policy derivatives,

$$B_M^{\text{foc}} = \sum_{j \neq i} \frac{\partial n_{j,t+1}}{\partial n_{it}}, \quad B_M^{\text{env}} = \sum_{j \neq i} \frac{\partial n_{jt}}{\partial n_{k,t-1}}, \quad k \neq i. \quad (\text{A.7})$$

The second includes the named rival k 's own response. For the recursive expression, set $k \neq i$ and temporarily define the stationary bracket

$$K_i = \beta \frac{\partial \Pi_i}{\partial n_k} - n \frac{\partial w_i}{\partial n_k} - \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q}.$$

The cross envelope is

$$\frac{\partial \Pi_i}{\partial n_k} = \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} (1 - \delta - p) + B_M^{\text{env}} K_i.$$

Substitute this into the definition of K_i to obtain

$$(1 - \beta B_M^{\text{env}})K_i = -n \frac{\partial w_i}{\partial n_k} - \frac{1}{M} \frac{\nu}{1 - \nu} \frac{\kappa}{q} [1 - \beta(1 - \delta - p)].$$

Multiplication by $\beta(1 - \chi)B_M^{\text{foc}}$ gives (A.8), whenever the denominator is nonzero. This elimination retains the named rival's own response in B_M^{env} .

$$S_M = -\frac{1}{M} \frac{\beta(1 - \chi)B_M^{\text{foc}}}{1 - \beta B_M^{\text{env}}} \left[N_M \frac{\partial w_i}{\partial n_k} + \frac{\nu}{1 - \nu} \frac{\kappa}{q} \{1 - \beta(1 - \delta - p)\} \right]. \quad (\text{A.8})$$

The numerator captures rivals' initial response and the denominator its recurrence through future states. Wage and recruiting costs determine the profit effect of that response.

A.6 Planner comparison

For a given aggregate transition,

$$V = \left(\frac{H}{\mu u^\nu} \right)^{1/(1-\nu)}, \quad \frac{\partial V}{\partial n_i} = \frac{1}{(1 - \nu)q}.$$

The planner envelope includes the benefit of surviving employment and the cost of a smaller future search pool. Substituting it into the planner first-order condition gives (26).

At a symmetric steady state, write

$$K = \frac{\kappa}{q} [1 - \beta(1 - \delta)], \quad L = \beta \kappa \theta.$$

Then the planner residual and decentralized residual are

$$\begin{aligned} F_M^P &= \alpha A_i n^{\alpha-1} - b - \frac{K + \nu L}{1 - \nu}, \\ F_M^D &= (1 - \chi)[\alpha A_i n^{\alpha-1} - b - b_0 + G_M^r] - K - \chi L - \frac{1}{M} \frac{\nu}{1 - \nu} (K + L). \end{aligned}$$

Subtract $(1 - \chi)F_M^P$. The coefficients of both K and L become

$$\frac{\nu(1 - 1/M) - \chi}{1 - \nu}.$$

Since $P_M = \nu(K + L)/(1 - \nu)$, this proves (30) and hence Proposition 2. Setting $G_M^r = 0$ gives the pure congestion condition (27). In the atomistic multiworker economy, G_M^r retains the canonical wage-bill term.

B Proof of local convergence

This appendix proves Proposition 1 from the exact equilibrium equations. Write $\varepsilon = 1/M$, $\eta = \nu/(1 - \nu)$, and μ_0 for matching efficiency. Let m denote a distribution of rival states.

B.1 Assumptions

Assumption 1 (Feasibility and local stability). Primitives satisfy $A, \kappa, \mu > 0$ and $\alpha, \beta, \chi, \nu, \delta \in (0, 1)$. The atomistic steady state and a surrounding real neighborhood satisfy the matching, positive-hiring, and bargaining restrictions with strict slack. In addition,

$$N^* < 1 - \delta, \quad (1 - \delta)\nu(1 - N^*) > \chi\delta N^*. \quad (\text{B.1})$$

The regular Stole–Zwiebel wage convention is maintained.

Assumption 2 (Regularity of the local equilibrium equations). In coordinates that keep a firm’s size relative to the symmetric firm fixed, the exact matching, wage, continuation, and policy equations are well defined and jointly smooth on fixed function spaces controlling their required state derivatives. Transition images remain strictly inside the domains on which these functions are defined. Appendix B states the analytic-domain version used in the proof.

Assumption 2 controls the equilibrium functions and their state derivatives on a common neighborhood as employer size changes.

B.2 Precise regularity condition and coordinates

Assumption 1 supplies strict real feasibility on a marked path containing the local aggregate transition and the entire own-employment segment used by (A.5). The precise analytic version of Assumption 2 is the following:

The exact insertion, deletion, pushforward, matching, regular wage, pair-envelope, and two-policy maps extend jointly analytically in ε and the unknown fields on fixed nested marked domains. Their transition images have uniform positive distances from the next domain boundaries. The associated maps are bounded between the fixed derivative-controlling spaces used below.

I refer to this analytic regularity condition as A3 below.

Scale employment by $x_i = Mn_i$. For a firm's own state x and the empirical distribution m of its $M - 1$ rivals, define

$$G_\varepsilon(x, m) = MN_i(\mathbf{n}), \quad \widehat{W}_\varepsilon(x, m) = w_i(\mathbf{n}).$$

For a named rival of state r , let ρ be the empirical distribution of the other $M - 2$ employers and define

$$P_\varepsilon(x, r, \rho) = M \frac{\partial \Pi_i(\mathbf{n})}{\partial n_k}, \quad k \neq i.$$

The regular wage is written

$$\widehat{W}_\varepsilon(x, m) = \frac{\chi \alpha A}{1 - \chi(1 - \alpha)} x^{\alpha-1} + V_\varepsilon(x, m). \quad (\text{B.2})$$

Only the remainder V_ε is required to be bounded at zero own employment. The production component has a known integrable payroll singularity.

Set $a_\varepsilon = \varepsilon/(1 - \varepsilon)$. The distribution of competitors faced by a rival at z is exactly

$$E_{a_\varepsilon}^x(z, m) = m + a_\varepsilon(\delta_x - \delta_z). \quad (\text{B.3})$$

Deleting that rival from m gives

$$R_{a_\varepsilon}(z, m) = \frac{m - a_\varepsilon \delta_z}{1 - a_\varepsilon}.$$

Given G , the transition of the empirical distribution is

$$\begin{aligned} x^+ &= G(x, m), & z^+ &= G(z, E_{a_\varepsilon}^x(z, m)), \\ (Q_{\varepsilon, G}^x m)[f] &= m[z \mapsto f(z^+)], & m^+ &= Q_{\varepsilon, G}^x m. \end{aligned} \quad (\text{B.4})$$

Aggregate inherited employment, the search pool, and gross hiring are

$$\begin{aligned} \bar{x} &= \varepsilon x + (1 - \varepsilon)m[z], & u &= 1 - \bar{x}, \\ r_i &= x^+ - (1 - \delta)x, & r(z) &= z^+ - (1 - \delta)z, \\ H &= \varepsilon r_i + (1 - \varepsilon)m[r]. \end{aligned} \quad (\text{B.5})$$

Matching then follows the original primitive formulas. The concentration factor in the alternative and wage equations is exactly

$$\mathcal{C} = 1 + \frac{\eta}{H^2} [\varepsilon^2 r_i^2 + \varepsilon(1 - \varepsilon)m[r^2]]. \quad (\text{B.6})$$

For pair states, insert the named rival and own state in the remaining-rival distribution before applying the same replacement and deletion operations. In particular,

$$m_{-i} = a_\varepsilon \delta_r + (1 - a_\varepsilon)\rho, \quad m_{-k} = a_\varepsilon \delta_x + (1 - a_\varepsilon)\rho.$$

The exact remaining-rival coefficients are

$$c_\varepsilon = \frac{1 - 2\varepsilon}{1 - \varepsilon}, \quad d_\varepsilon = \frac{1 - 2\varepsilon}{(1 - \varepsilon)^2}. \quad (\text{B.7})$$

They follow from $(M - 2)a_\varepsilon = c_\varepsilon$ and $(M - 2)a_\varepsilon^2/\varepsilon = d_\varepsilon$. Their limits at zero are analytic, with same-rival and distinct-rival responses represented separately throughout.

For a fixed outer one-firm complex domain Ω , take the dual of its disc algebra as the distribution space. Evaluation at z is the Dirac functional δ_z . On a domain at distance at least $s > 0$ from the boundary, Cauchy's formula gives

$$\|\delta_z^{(k)}\| \leq k!/s^k. \quad (\text{B.8})$$

Use bounded holomorphic fields on three fixed nested own/distribution domains and the corresponding pair domains. Positive boundary buffers control each marked-state derivative under restriction, $\|DF\|_{j+1} \leq C_s \|F\|_j$. A3 provides bounded composition and population maps on these domains.

B.3 The atomistic stable path

For aggregate transitions $x \mapsto y \mapsto z$, set

$$H(x, y) = y - (1 - \delta)x, \quad u(x) = 1 - x,$$

and let $q(x, y)$ and $\theta(x, y)$ be the associated matching rates. Define

$$\begin{aligned} \mathcal{R}(x, y, z) = & ay^{\alpha-1} - (1 - \chi)(b + b_0) - \frac{\kappa}{q(x, y)} \\ & + \beta(1 - \delta)\frac{\kappa}{q(y, z)} - \chi\beta\kappa\theta(y, z), \quad a = \frac{(1 - \chi)\alpha A}{1 - \chi(1 - \alpha)}. \end{aligned} \quad (\text{B.9})$$

The stationary residual is $F_\infty(N) = \mathcal{R}(N, N, N)$. Direct differentiation gives

$$\begin{aligned} F'_\infty(N) = & a(\alpha - 1)N^{\alpha-2} - [1 - \beta(1 - \delta)]\frac{\kappa}{q(N)}\frac{\nu}{(1 - \nu)N(1 - N)} \\ & - \chi\beta\kappa\theta(N)\frac{1}{(1 - \nu)N(1 - N)} < 0. \end{aligned} \quad (\text{B.10})$$

Together with the boundary signs, this proves the unique atomistic root. Let R_x, R_y, R_z be derivatives of $\mathcal{R}$ at (N^*, N^*, N^*) . Two derivatives have useful primitive expressions:

$$R_x = \frac{\kappa}{q^*} \frac{\nu}{1 - \nu} \frac{1 - \delta - N^*}{\delta N^*(1 - N^*)} > 0, \quad (\text{B.11})$$

$$R_z = \frac{\beta\kappa}{q^*} \frac{(1 - \delta)\nu(1 - N^*) - \chi\delta N^*}{(1 - \nu)\delta N^*(1 - N^*)} > 0. \quad (\text{B.12})$$

The inequalities follow from (B.1), while $R_x + R_y + R_z = F'_\infty(N^*) < 0$. Therefore the polynomial $Q(\lambda) = R_z\lambda^2 + R_y\lambda + R_x$ satisfies $Q(0) > 0$, $Q(1) < 0$, and opens upward. Its two real roots obey

$$0 < \lambda_- < 1 < \lambda_+. \quad (\text{B.13})$$

Since $R_z \neq 0$, the implicit-function theorem writes the second-order equation as a two-dimensional transition map. The local stable-manifold theorem then supplies an analytic graph

$$N_{t+1} = g(N_t), \quad g(N^*) = N^*, \quad g'(N^*) = \lambda_-. \quad (\text{B.14})$$

This is the unique nearby graph whose forward paths remain in the local region and converge to N^* . The second direction expands and is excluded by local boundedness. Vieta's relation also gives

$$R_y + R_z\lambda_- = -R_x\lambda_+ \neq 0. \quad (\text{B.15})$$

B.4 Verification at the atomistic endpoint

Let $K = m[z]$. At zero own mass, an individual target y does not affect aggregate current and future targets Y, Z . The alternative residual becomes

$$\mathfrak{R}(y; K, Y, Z) = ay^{\alpha-1} - (1 - \chi)(b + b_0) - \frac{\kappa}{q(K, Y)} + \beta(1 - \delta)\frac{\kappa}{q(Y, Z)} - \chi\beta\kappa\theta(Y, Z). \quad (\text{B.16})$$

Its derivative in y is strictly negative. Setting $Y = g(K)$ and $Z = g(Y)$ gives zero at $y = g(K)$ by (B.9), so the full endpoint policy is

$$G_0(x, m) = g(K). \quad (\text{B.17})$$

Every rival then chooses $g(K)$ and the next distribution is $\delta_{g(K)}$. The regular wage remainder is

$$V_0(x, m) = (1 - \chi)(b + b_0) + \chi\beta\kappa\theta(K, g(K)), \quad (\text{B.18})$$

independent of own employment x . Substitution into (A.3) gives (B.16). The pair-envelope field is the unique solution of the discounted linear recursion specified next. These fields solve the full equilibrium equations at the atomistic endpoint.

B.5 Solving the wage and cross-envelope equations

For a policy G close to G_0 , the regular wage integral and the exact pair envelope have block form

$$V = B_\varepsilon^W(G) + \varepsilon L_\varepsilon^{WP}(G)P + \varepsilon K_\varepsilon^W(G)V, \quad (\text{B.19})$$

$$P = B_\varepsilon^P(G) + L_\varepsilon^{PW}(G)V + K_\varepsilon^P(G)P. \quad (\text{B.20})$$

To derive their coefficients, in (B.4) define

$$A_i(z) = D_m G(z, E_{a_\varepsilon}^x(z, m))[\delta'_x], \quad D_i = r_i \eta \frac{\kappa}{qH}.$$

Let ρ_z^+ delete the named rival z^+ from m^+ . The wage operators are

$$B_\varepsilon^W = \mathcal{I}_\chi \left[(1 - \chi)(b + b_0) - \chi\beta\varepsilon r_i \eta \frac{\kappa}{qu} + \chi\beta\kappa\theta\mathcal{C} - \varepsilon\chi\beta\langle m, A_i D_i \rangle \right], \quad (\text{B.21})$$

$$L_\varepsilon^{WP}P = \mathcal{I}_\chi \left[\chi\beta^2\langle m, A_i(z)P(x^+, z^+, \rho_z^+) \rangle \right], \quad (\text{B.22})$$

$$K_\varepsilon^W V = \mathcal{I}_\chi \left[-\frac{\chi\beta}{1-\varepsilon} \langle m, A_i(z) x^+ D_m V(x^+, m^+) [\delta'_{z+}] \rangle \right]. \quad (\text{B.23})$$

At a pair state, let $A_k = \partial_x G(r, m_{-k})$. For a remaining rival z , let $A_j(z)$ be its distribution derivative in the direction δ'_r after exact replacement by the own and named states. The pair operators are

$$B_\varepsilon^P = -r_i \eta \frac{\kappa}{q} \left(\frac{1}{u} - \frac{1-\delta}{H} \right) - A_k D_i - c_\varepsilon \langle \rho, A_j D_i \rangle, \quad (\text{B.24})$$

$$\begin{aligned} L_\varepsilon^{PW} V = & -\frac{A_k}{1-\varepsilon} x^+ D_m V(x^+, m_{-i}^+) [\delta'_{r+}] \\ & - d_\varepsilon \langle \rho, A_j(z) x^+ D_m V(x^+, m_{-i}^+) [\delta'_{z+}] \rangle, \end{aligned} \quad (\text{B.25})$$

$$K_\varepsilon^P P = \beta A_k P(x^+, r^+, \rho^+) + \beta c_\varepsilon \langle \rho, A_j(z) P(x^+, z^+, \rho_{ij}^+) \rangle. \quad (\text{B.26})$$

All next distributions in these equations are exact empirical deletions and pushforwards. The coefficients in (B.7) give the named-versus-remaining-rival counts. In particular, the order-one wage-to-pair term is retained. Its return to the wage equation carries the explicit ε in (B.19).

Under A3, the six operators are bounded and jointly analytic on fixed buffered domains. Strict feasibility bounds u, H, q and their required reciprocals. The regular wage integral is bounded by (A.5), while each state derivative uses a fixed Cauchy buffer.

At $(0, G_0)$, $A_k = 0$, $A_j(z) = g'(K)$, and

$$(K_0^P P)(x, r, \rho) = \beta g'(K) P(g(K), g(K), \delta_{g(K)}). \quad (\text{B.27})$$

Shrink the local domain so that $\beta \sup |g'| < 1$. The operator $I - K_0^P$ then has a bounded inverse by its Neumann series. The block derivative in (V, P) is triangular at the endpoint:

$$I - \begin{pmatrix} 0 & 0 \\ L_0^{PW} & K_0^P \end{pmatrix}.$$

Its inverse is bounded; it first solves the wage row and then the pair row. Continuity first preserves this inverse at $\varepsilon = 0$ for G in a sufficiently small fixed ball around G_0 . The finite- ε block differs from the zero- ε block at the *same* G by $O(\varepsilon)$ uniformly on that ball. The resolvent identity therefore yields a bounded inverse for small ε and unique fields $V_\varepsilon[G], P_\varepsilon[G]$. On an inner domain,

$$\|V_\varepsilon[G] - V_0[G]\| + \|P_\varepsilon[G] - P_0[G]\| \leq C|\varepsilon|. \quad (\text{B.28})$$

Analyticity on a fixed outer policy ball gives the same order for derivatives with respect to G on a smaller ball. This controls the potentially amplifying wage–continuation feedback before solving for the hiring policy.

B.6 The hiring-policy inverse and contraction

Keep current and continuation policies distinct. Starting from (x, m) , use U for the first transition and G for the second. Index their matching objects by 0 and 1. After eliminating V, P , the exact alternative residual is

$$\begin{aligned} \mathcal{C}_\varepsilon(U; G) = & \frac{1-\chi}{\chi} [\widehat{W}_\varepsilon[G](x_0^+, m_0^+) - b - b_0] \\ & - \frac{\kappa}{q_0} \left(1 + \eta\varepsilon \frac{r_0}{H_0}\right) + \beta(1-\delta) \frac{\kappa}{q_1} \left(1 + \eta\varepsilon \frac{r_1}{H_1}\right) - \beta\kappa\theta_1\mathcal{C}_1. \end{aligned} \quad (\text{B.29})$$

At $U = G$ this is the original alternative row. At the endpoint set $Y = g(K)$, $Z = g(Y)$. If v perturbs U and h perturbs G , write $v_x = v(x, m)$, $\bar{v} = m[v(\cdot, m)]$, and $h_1 = h(Y, \delta_Y)$. The first next distribution changes by $\bar{v}\delta'_Y$, and the second aggregate target changes by $h_1 + g'(Y)\bar{v}$.

Let $\mathfrak{A}, \mathfrak{C}, \mathfrak{D}$ be the derivatives of (B.16) with respect to y, Y, Z . Then

$$\mathfrak{A} = a(\alpha - 1)g(K)^{\alpha-2} < 0, \quad \mathfrak{A} + \mathfrak{C} = R_y(K), \quad \mathfrak{D} = R_z(K).$$

The linearized equation is

$$\mathfrak{A}v_x + [\mathfrak{C} + \mathfrak{D}g'(Y)]\bar{v} + \mathfrak{D}h_1 = 0.$$

Averaging gives $\Delta(K)\bar{v} + \mathfrak{D}h_1 = 0$, where

$$\Delta(K) = R_y(K) + R_z(K)g'(g(K)).$$

Subtracting the average leaves $\mathfrak{A}(v_x - \bar{v}) = 0$. Thus the centered and mean responses are both uniquely determined. In operator form, for the projection $\mathcal{P}H = m[H(\cdot, m)]$ with $\mathcal{P}^2 = \mathcal{P}$,

$$\begin{aligned} D_U\mathcal{C}_0 &= \mathfrak{A}(I - \mathcal{P}) + \Delta\mathcal{P}, \\ (D_U\mathcal{C}_0)^{-1} &= \mathfrak{A}^{-1}(I - \mathcal{P}) + \Delta^{-1}\mathcal{P}. \end{aligned} \quad (\text{B.30})$$

The second inverse exists since $\Delta(N^*) = -R_z\lambda_+ \neq 0$ by (B.15); continuity bounds it away from zero nearby. Hence the inverse is derived from the primitive root ordering and own marginal-product curvature.

The implicit-function theorem defines $T_\varepsilon(G) = U$ by $\mathcal{C}_\varepsilon(U; G) = 0$. Its endpoint derivative is

$$(DT_0[G_0]h)(x, m) = -\frac{R_z(K)}{R_y(K) + R_z(K)g'(g(K))}h(g(K), \delta_{g(K)}). \quad (\text{B.31})$$

At steady state the multiplier equals $1/\lambda_+ < 1$. On a sufficiently small fixed domain and policy ball its operator norm is therefore bounded by some $\rho < 1$. Equations (B.28) and A3 give $T_\varepsilon - T_0 = O(\varepsilon)$ together with the same derivative bound on a smaller ball. Choosing that ball and then ε_0 small enough makes T_ε a contraction mapping the ball into itself. Banach's fixed-point theorem gives a unique G_ε there and

$$\|G_\varepsilon - G_0\| \leq \frac{\|T_\varepsilon(G_0) - T_0(G_0)\|}{1 - \rho} \leq C|\varepsilon|.$$

Together with the solved wage and pair block and one final domain restriction, this yields

$$\|G_\varepsilon - G_0\|_{C^1} + \|V_\varepsilon - V_0\|_{C^1} + \|P_\varepsilon - P_0\|_{C^1} \leq C|\varepsilon|. \quad (\text{B.32})$$

The derivative norm is taken on the fixed inner marked domain.

B.7 Stationary values and the vanishing incentives

On the symmetric restriction the stationary equation is

$$\Psi(\varepsilon, N) = G_\varepsilon(N, \delta_N) - N = 0.$$

At the endpoint its derivative in N is $\lambda_- - 1 \neq 0$. The finite-dimensional implicit-function theorem and (B.32) imply a unique nearby N_ε and the employment and wage bound (23).

The economically relevant original-coordinate derivatives satisfy exact identities. At symmetry,

$$B_M^{\text{foc}} = D_m G_\varepsilon(N, \delta_N)[\delta'_N], \quad (\text{B.33})$$

$$M \frac{\partial \Pi_i}{\partial n_k} = P_\varepsilon(N, N, \delta_N), \quad (\text{B.34})$$

$$\frac{\partial w_i}{\partial n_k} = \frac{1}{1 - \varepsilon} D_m V_\varepsilon(N, \delta_N)[\delta'_N], \quad k \neq i, \quad (\text{B.35})$$

$$\left. \frac{\partial w_i}{\partial n_i} - \frac{\partial w_i}{\partial n_i} \right|_{\text{SZ}} = M \partial_x V_\varepsilon(N, \delta_N). \quad (\text{B.36})$$

The first three are uniformly bounded by (B.32). For the fourth, (B.18) gives $\partial_x V_0 = 0$, so $\partial_x V_\varepsilon = O(\varepsilon)$ and the original-coordinate difference is also bounded.

Consequently,

$$\begin{aligned} \Xi_M &= B_M^{\text{foc}} \left[\beta M \frac{\partial \Pi_i}{\partial n_k} - N_M \frac{\partial w_i}{\partial n_k} - \eta \frac{\kappa}{q} \right] = O(1), \\ \Upsilon_M &= N_M \left[\frac{\partial w_i}{\partial n_i} - \frac{\partial w_i}{\partial n_i} \right]_{\text{SZ}} = O(1). \end{aligned}$$

The strategic term is $\beta(1 - \chi)\Xi_M/M$ and the additional wage term is $-(1 - \chi)\Upsilon_M/M$. Together with (20), these bounds prove that all additional Euler contributions vanish at order $1/M$.

B.8 Best responses

Reconstruct the scaled firm value $\mathcal{U}_\varepsilon = M\Pi_i$ by discounted policy evaluation on a fixed real invariant tube. The evaluation map is a contraction with modulus β . At a current unilateral target deviation y , keep rivals' targets fixed and consider

$$\widehat{J}_\varepsilon(y; s) = Ay^\alpha - \widehat{W}_\varepsilon(y, m_{-i}^+)y - \kappa \frac{y - (1 - \delta)x}{q_\varepsilon(y; s)} + \beta \mathcal{U}_\varepsilon(y, m_{-i}^+). \quad (\text{B.37})$$

Its exact derivative is the raw firm first-order condition. Substituting the bargaining and envelope identities makes it identical to (B.29) at the candidate policy. Differentiated policy evaluation recovers the same own and cross envelopes: the own-policy derivative is multiplied by the zero firm residual, and the remaining terms are precisely (15). The bounded linear envelope recursion is uniquely solved in the spaces above, so the reconstructed derivative is the pair field already used in the construction.

At zero own mass,

$$\partial_y \widehat{J}_0(y; s) = ay^{\alpha-1} + L_0(s), \quad \partial_{yy} \widehat{J}_0(y; s) = a(\alpha - 1)y^{\alpha-2} < 0.$$

On a fixed feasible marked action interval, strict feasibility and A3 make the finite- ε derivative uniformly C^1 -close to this one. Its second derivative is therefore strictly negative for sufficiently small ε . The constructed interior target is its unique zero and hence the strict maximizer over all feasible deviations remaining in the interval. The invariant-domain policy

evaluation supplies the associated continuation values. This proves the local best-response statement.

To extend the construction to unrestricted deviations, suppose continuation values are globally defined and there is a fixed cutoff $\bar{Y}$ and a positive gap ζ such that

$$\hat{J}_\varepsilon(G_\varepsilon(s); s) \geq \sup_{\substack{y > \bar{Y} \\ y \text{ feasible}}} \hat{J}_\varepsilon(y; s) + \zeta \quad (\text{B.38})$$

uniformly over the relevant states and small ε . Together with a global extension supporting policy evaluation and optimality over the remaining bounded deviations, this tail condition is sufficient for an unrestricted MPE. Proposition 1 establishes best responses on the local invariant neighborhood. $\square$

C Numerical representation and diagnostics

C.1 Job finding and vacancy filling

The stationary identity $H = \delta N$ and matching feasibility determine the worker and firm contact probabilities at each equilibrium. Table 10 reports them at the weekly frequency used in calibration. With separations fixed at 0.48 percent per week, concentration changes unemployment through job finding.

Table 10: Worker and employer matching probabilities

M	Unemployment (%)	Job finding (%)	Vacancy filling (%)
1	9.09	4.80	16.55
2	7.64	5.80	13.69
4	6.83	6.55	12.13
10	6.33	7.11	11.18
20	6.14	7.34	10.82
∞	5.52	8.22	9.66

Notes: Job finding is $\delta N/(1 - N)$; vacancy filling is $\mu\theta^{-\nu}$. Probabilities are weekly.

C.2 Equilibrium objects and residuals

The linear model is represented by hiring policies $\mathcal{N}_i(\mathbf{k})$, wage schedules $w_i(\mathbf{n})$, and cross values $\partial\Pi_i(\mathbf{k})/\partial k_j$ for $j \neq i$. At every evaluation point, form the two transitions $\mathbf{k} \mapsto \mathbf{n} =$

$\mathcal{N}(\mathbf{k}) \mapsto \mathbf{n}^+ = \mathcal{N}(\mathbf{n})$. The firm row is (A.3), the wage row is (A.4), and the pair row is the cross part of (15). The own envelope is reconstructed from that same equation. The raw first-order condition provides an independent algebraic check.

The CRR system adds the common search policy $\mathcal{E}(\mathbf{k})$ and replaces the linear worker equation by the surplus and nonlinear bargaining equations in Appendix E. Search derivatives enter the envelope through the inherited state; current effort is predetermined in the target choice.

C.3 Representing employer symmetry

Let $\Delta_j = k_j - \bar{n}$. A firm's policy and wage are invariant to permutations of rival labels. Their degree- d terms can be indexed by (a, λ) , where a is the own-coordinate derivative order and $\lambda = (\lambda_1, \dots, \lambda_L)$ is an integer partition describing the orders attached to L distinct rivals. The restrictions are $a + |\lambda| \leq d$ and $L \leq M - 1$. Use monomials

$$\Phi_{a,\lambda}^i(\Delta) = \frac{\Delta_i^a}{a! \prod_{r=1}^L \lambda_r!} \sum_{\substack{\text{distinct rival labels } j_1, \dots, j_L \\ \text{unique assignments}}} \prod_{r=1}^L \Delta_{j_r}^{\lambda_r}. \quad (\text{C.1})$$

Equal parts of the partition do not generate duplicate assignments. With this convention each coefficient is the corresponding labeled derivative at the center. For example,

$$\begin{aligned} \mathcal{N}_i(\mathbf{k}) = & \bar{n} + c_o \Delta_i + c_r \sum_{j \neq i} \Delta_j + \frac{c_{oo}}{2} \Delta_i^2 + c_{or} \Delta_i \sum_{j \neq i} \Delta_j \\ & + \frac{c_{rr}}{2} \sum_{j \neq i} \Delta_j^2 + c_{rs} \sum_{j < k; j, k \neq i} \Delta_j \Delta_k + \dots \end{aligned} \quad (\text{C.2})$$

The coefficients c_{rr} and c_{rs} separately capture repeated responses to one rival and responses to two distinct rivals.

The pair envelope distinguishes firm i and a named rival j . Its terms are indexed by (a, b, λ) , where a, b count derivatives with respect to those two states and λ describes the remaining rivals. The restrictions become $a + b + |\lambda| \leq d$ and $L \leq M - 2$. A complete degree- d approximation includes all terms satisfying these restrictions.

Table 11: Size of the complete degree-five symmetric system

M	1	2	3	4	5	6	≥ 7
Terms in each one-firm family	6	21	34	41	44	45	45
Terms in the pair family	0	21	56	80	91	95	96
Total unknowns	12	63	124	162	179	185	186

Notes: The two one-firm families are hiring and wages. Their constant terms include steady-state employment and the wage. The pair family represents the cross envelope. Terms requiring too many distinct employers are absent in small topologies.

C.4 Polynomial evaluation and nonlinear solution

The implementation propagates truncated multivariate polynomial coefficients. Products use coefficient convolution, and a power is evaluated using

$$X^r = X_0^r \sum_{s=0}^d \binom{r}{s} \left(\frac{X - X_0}{X_0} \right)^s \quad \text{with terms above degree } d \text{ discarded.}$$

This handles the fractional powers in production and matching. Composition substitutes the policy polynomials into one another. At degree d , at most $d + 2$ coordinates must be separately marked for a pair residual. Other employers enter sums with exact multiplicities for each derivative type.

The coefficient of each admissible term in every residual is set to zero, with coefficients above degree d set to zero as the terminal closure. Comparisons across degrees measure sensitivity to this truncation.

Unknowns and residuals are scaled to responses to relative employment changes. For derivative order r , representative unknown normalizations are

$$N = M\bar{n}, \quad \tilde{c}_r^N = c_r^N \bar{n}^{r-1}, \quad \tilde{c}_r^w = c_r^w \bar{n}^r / \bar{w}, \quad \tilde{c}_r^P = M c_r^P \bar{n}^r / \bar{w}.$$

These transformations preserve the roots of the equilibrium system. Nonlinear solves use damped or trust-region steps, with row and column scaling of the coefficient Jacobian. Lower-degree solutions and nearby parameter or policy solutions supply initial conditions. Diagnostics include independent residual evaluation, alternative starts, and adjacent-order comparisons, as reported below. The atomistic benchmark is solved directly from its limiting equations.

C.5 Numerical diagnostics

I check equation residuals, reconstructed accounting identities, agreement between equivalent first-order conditions, and feasibility. Residuals measure how closely the polynomial satisfies the coefficient equations. The Euler and wage decompositions provide independent accounting checks. Feasibility checks cover employment, hiring, matching probabilities, consumption, and the relevant local response blocks.

For the twelve baseline finite-market points, the maximum absolute Euler closure error is 4.30×10^{-13} . The maximum wage-level decomposition error, including the atomistic benchmark, is 3.78×10^{-13} . The 44 CRRA profit-share policy optima have a maximum equilibrium residual below 2.51×10^{-10} . The UI-derivative calculations, evaluated also at zero policy, have a maximum center residual below 8.10×10^{-8} and a channel-closure discrepancy below 1.44×10^{-7} .

The decomposition in Figure 7 uses the same degree-five roots at $M = 1, 2, 4, 10$. The worker and firm tightness derivatives are combined to match (A.6). Multiplying each additional slope component by $-(1 - \chi)n$ gives its Euler contribution. Their sum reproduces the GE wage-bill effect to within 4.7×10^{-15} . The unconverted slope accounting closes to within 9.4×10^{-15} . The tightness and hiring-share components are also reconstructed directly from the policy derivatives.

The degree-five Hosios calculations include independent residual checks and alternative starts. Their coefficient Jacobians are poorly conditioned in some markets: the augmented Jacobian condition number is about 3.0×10^{10} at $M = 2$. For the CRRA UI schedule, an approximation-order comparison is available at the duopoly optimum with $\phi = 1$. The optimal increment is -0.023500 at degree three and -0.023004 at degree five.

C.6 Adjustment of the employment distribution

At a symmetric steady state, write the own policy derivative as a and each rival derivative as d . The local transition matrix has diagonal a and off-diagonal d . An aggregate perturbation therefore evolves at rate $a + (M - 1)d$, while any zero-sum redistribution of employment evolves at rate $a - d$. Table 13 computes the half-life of each disturbance from these rates.

Figure 13 starts from an additional infinitesimal amount of employment at one employer, with other employers initially unchanged. If $r_A = a + (M - 1)d$ and $r_D = a - d$, the

Table 12: Numerical specifications

Exercise	Finite markets	Atomistic benchmark
Baseline levels and Euler/wage decomposition	Complete degree five, twelve M values through 30	Exact scalar endpoint
Planner-implementing bargaining	Degree five for $M = 2, \dots, 10$; analytic monopoly boundary	Exact scalar endpoint
CRRA profit-income/UI schedules	Complete degree three, $M = 1, \dots, 10$	Exact marked-wage-ODE endpoint
UI welfare derivative	Complete degree three at selected M and profit-income arrangements	Exact endpoint response system
Optimal common UI	Complete degree three; fixed degree-five calibration	Exact scalar endpoint
Merger remedies	Complete degree-five dominant-fringe system	Not used

Table 13: Persistence of aggregate employment and its distribution

M	Weekly persistence		Half-life (weeks)	
	Aggregate	Distribution	Aggregate	Distribution
2	0.9363	0.9853	10.5	46.9
4	0.9289	0.9788	9.4	32.4
10	0.9235	0.9668	8.7	20.5
30	0.9204	0.9433	8.4	11.9

Notes: Policy derivatives come from the accepted degree-five equilibrium at each market structure. For a persistence coefficient $r \in (0, 1)$, the half-life is $\log(1/2)/\log r$.

own-employer response after t weeks, per initial additional job, is

$$\frac{r_A^t}{M} + \left(1 - \frac{1}{M}\right) r_D^t.$$

The combined response of the other employers is $(1 - 1/M)(r_A^t - r_D^t)$, and aggregate employment changes by r_A^t . These paths are the linear response of the full Markov equilibrium to the inherited employment perturbation. They retain the same hiring and bargaining policies throughout.

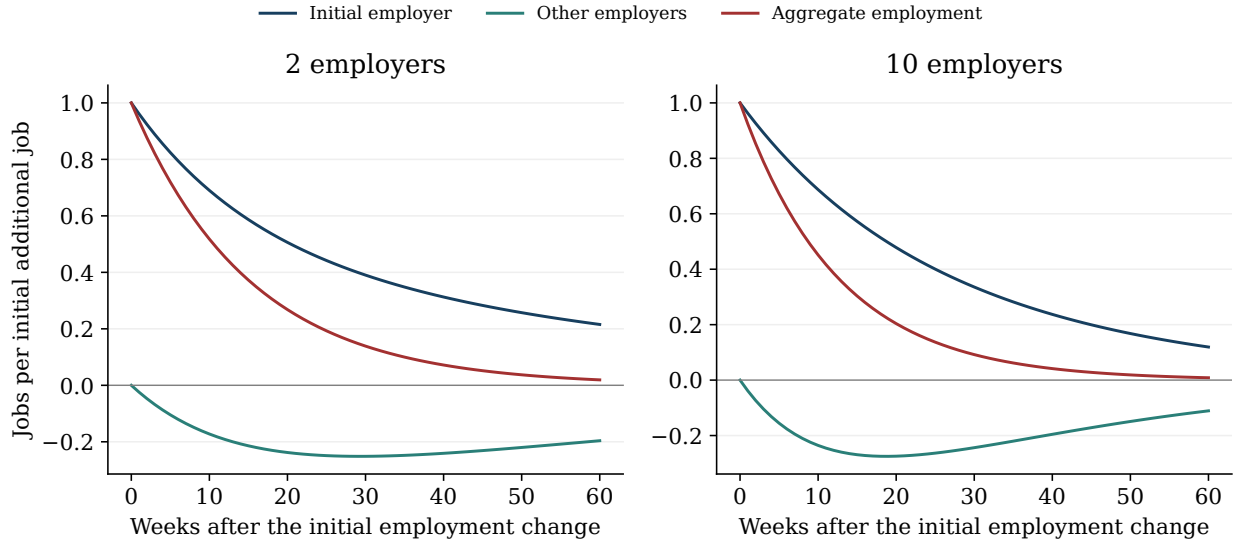


Figure 13: Employment adjustment following a change at one employer

Notes: First-order conditional transition paths, normalized by the initial additional employment at one employer. Other employers' initial employment is unchanged. Responses are computed from powers of the full equilibrium policy Jacobian.

C.7 Common and market-specific UI

The implementation condition varies by market when bargaining power is fixed and UI is the instrument. I now choose one common increment to maximize labor-force-weighted stationary welfare,

$$\max_{b_0} \sum_g \omega_g R(N_g(b_0)). \quad (\text{C.3})$$

Market weights and structural parameters remain at their baseline calibrated values. Each candidate policy requires solving every market's hiring, wage, and continuation equations. The baseline and policy allocations in this comparison use the same approximation order.

The marginal welfare condition weights each market’s policy response by the social value of an additional job:

$$\sum_g \omega_g R_N(N_g(b_0)) \frac{dN_g}{db_0} = 0. \quad (\text{C.4})$$

A common policy balances the cost of reducing employment in underhiring markets against the benefit of reducing excessive employment elsewhere. Market-specific transfers can instead bring every market to the stationary-welfare maximum, $N = 0.932137$, with welfare 0.96797677.

Table 14: Common and market-specific UI

Policy	b_0	Unemployment (%)	Welfare	Gain recovered (%)
No UI increment	0.0000	6.02	0.96750256	0.0
Aggregate employment target	0.0802	6.79	0.96776083	54.5
Optimal common UI	0.0802	6.79	0.96776083	54.5
Market-specific UI	Varies	6.79	0.96797677	100.0

Notes: All rows use the same degree-three finite-market system, the exact atomistic endpoint, and fixed baseline parameters and weights. The gain is measured relative to the no-increment allocation computed at that same degree. The common-policy search is over $b_0 \in [-0.10, 0.20]$; its optimum is interior. Market-specific transfers solve each market to the common stationary-welfare maximum. The aggregate-employment rule targets that same employment level in the weighted economy.

The optimal common increment is 0.0802 and recovers 54.5 percent of the stationary welfare gain available under market-specific transfers. A common policy chosen to match aggregate employment to the stationary-welfare target is almost identical in this calibration. The remaining welfare difference reflects the allocation of employment across markets. A uniform increase in the outside option reduces hiring in concentrated markets as well as in markets with excessive employment, whereas the market-specific policy moves them in opposite directions. The common-policy search uses a grid over $b_0 \in [-0.1, 0.2]$, followed by scalar maximization and checks of the first and second derivatives. Table 15 compares the results across approximation orders, evaluating the baseline and policy allocations at the same order in each case. The share of the available welfare gain recovered by common UI rises from 44.8 percent at degree two to 54.5 percent at degree three. Welfare levels differ little across orders, but the gain ratio is more sensitive because the available welfare improvement is small.

Table 15: Approximation order in the common-UI exercise

Degree	Optimal b_0	Baseline welfare	Optimal welfare	Gain recovered (%)
2	0.07160	0.96750574	0.96771692	44.8
3	0.08019	0.96750256	0.96776083	54.5

Notes: Gain recovered is the common-policy welfare gain divided by the market-specific-policy gain, using the same-order baseline in both. All cases use the exact atomistic endpoint. The degree-three optimum has a welfare derivative below 2×10^{-9} in absolute value and a negative second derivative. Perturbed-root restarts and independently computed employment derivatives give the same local solution.

C.8 A Shapley decomposition of equilibrium outcomes

To measure how employment and wages respond to each incentive, I remove selected terms and solve the equilibrium again. I consider three groups: finite-employer recruiting internalization (T), future rival-policy responses (S), and the additional own-employment wage slope beyond the Stole–Zwiebel component (W). The recruiting switch changes the finite hiring-share terms wherever they enter, including the strategic expression; matching technology itself is unchanged. The wage switch replaces the own wage slope entering bargaining by its Stole–Zwiebel component. Wage levels and cross-employment wage responses continue to adjust in equilibrium.

For an outcome $y(C)$ obtained with the set of active channels C , the Shapley contribution of channel j averages its incremental effect over all orders in which the channels can be introduced:

$$\mathcal{S}_j(y) = \sum_{C \subseteq \{T, S, W\} \setminus \{j\}} \frac{|C|! (2 - |C|)!}{3!} [y(C \cup \{j\}) - y(C)]. \quad (\text{C.5})$$

The contributions sum to the difference between the full-channel and no-channel allocations. The latter has the atomistic symmetric employment and wage levels.

The intermediate economies can have multiple locally stable solutions. In the four-employer exercise, continuation from the no-channel and full-channel allocations selects different solutions when only S , or S and W , are active. Table 16 reports the attributions under these two selections, holding the other six allocations fixed. Both selections give the same total change, but allocate it differently across channels.

Recruiting internalization and the additional wage-bill incentive raise unemployment under both selections, while strategic interaction partly offsets them. Their quantitative shares depend on equilibrium selection in the intermediate economies. The initial degree-five exercise also found distinct stable solutions in the strategic-only economy; the full-seeded SW solution could not be carried to degree five at the required accuracy. The table therefore uses degree four throughout. The Euler decomposition holds the full equilibrium allocation

Table 16: Shapley contributions under two equilibrium selections

	Recruiting	Strategic	Wage-bill	Total
<i>Unemployment: percentage points</i>				
Selection A	+0.947	−0.204	+0.653	+1.395
Selection B	+1.179	−0.387	+0.604	+1.395
<i>Wage: level changes</i>				
Selection A	−0.0282	+0.0238	−0.0749	−0.0793
Selection B	−0.0596	+0.0499	−0.0696	−0.0793

Notes: Four employers, baseline primitives, and a common degree-four approximation for all intermediate economies and the full-channel allocation. Selection A uses the no-channel-seeded solutions for S and SW ; selection B uses the full-channel-seeded solutions. The other allocations coincide. All reported solutions are locally stable. Contributions add before rounding.

fixed. Here, employment and wages adjust as each incentive is removed, and the attribution depends on which intermediate equilibrium is selected.

D Empirical motivation and evidence

D.1 Employer concentration and unemployment

The empirical analysis examines the relationship between employer concentration and unemployment across commuting zones. It combines annual unemployment from the Local Area Unemployment Statistics with a concentration proxy constructed from County Business Patterns establishment-size bins. The sample contains 7,000 commuting-zone-year observations in 586 commuting zones. Unemployment is measured over 2005–2016 and concentration over 2004–2015.

The estimating equation is

$$U_{zt} = \beta_H \log(HHI_{z,t-1}) + \alpha_z + \gamma_t + \epsilon_{zt}, \quad (\text{D.1})$$

where U_{zt} is the unemployment rate in percentage points. The regression includes commuting-zone and year fixed effects, uses labor-force weights, and clusters standard errors by commuting zone. A 10 percent increase in concentration is associated with 0.082 percentage point higher unemployment, with a standard error of 0.036 and a p -value of 0.022. The graph displays weighted means in 15 bins of residualized log concentration; the fitted line comes from the full observation-level regression.

The concentration measure groups establishments by size, so establishments belonging to the same employer are counted separately. The estimated association is smaller and less precise without labor-force weights, with commuting-zone trends, or after trimming concentration tails. Future concentration is also positively associated with current unemployment. Section D.5 reports these specifications.

D.2 Job finding and the unemployment margin

Evidence on worker flows connects concentration more directly to recruiting. Using Norwegian administrative data, Berger et al. (2024) find lower unemployment-to-employment transition rates in more concentrated occupation–commuting-zone markets. In their within-market specification with controls, a one-standard-deviation increase in employment concentration is associated with a 0.77 percentage-point decline in the monthly job-finding rate, compared with a sample mean of 8.08 percent. The corresponding across-region estimate is

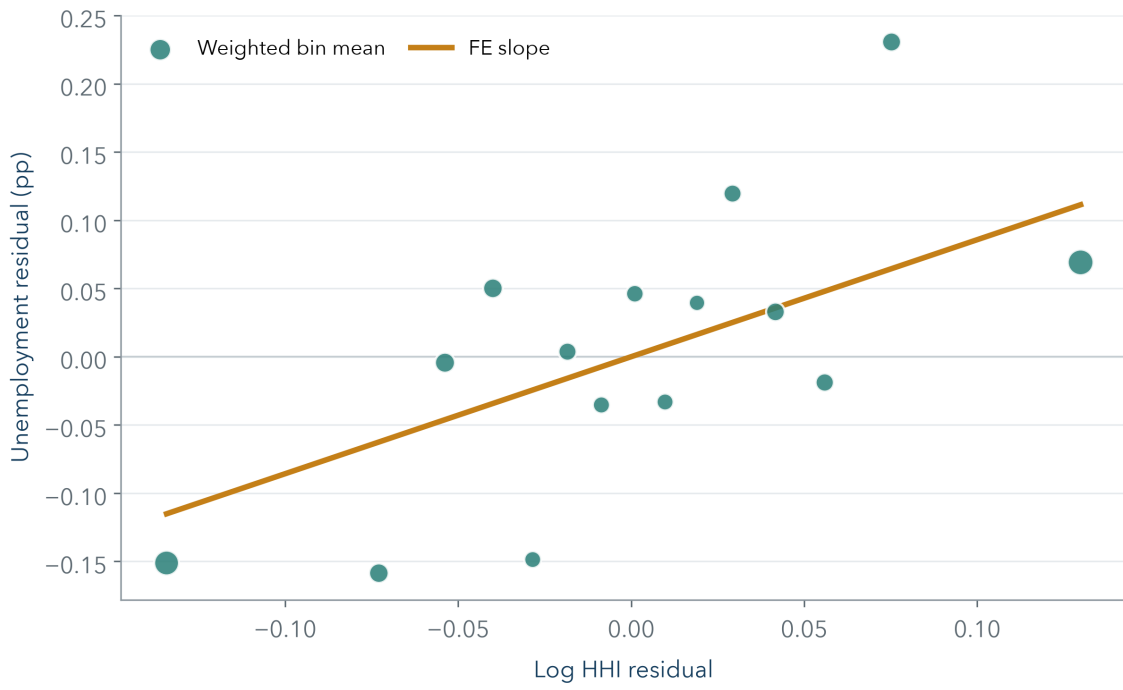


Figure 14: Employer concentration and unemployment

Notes: Both variables are residualized using commuting-zone and year fixed effects and LAUS labor-force weights. Bubble area reflects the labor force represented by each bin. Concentration is a lagged establishment-size-bin proxy. Sections D.3–D.5 describe construction, inference, and robustness.

0.89 percentage point. Separation rates also decline with concentration, and the unemployment association depends on the specification. These are descriptive flow relationships.³

The model studies the job-creation side of this evidence. With a fixed separation probability, the stationary employment condition is $p(1 - N) = \delta N$. Employer concentration affects unemployment through the job-finding probability p , whose level is determined by vacancy creation and matching. Its comparative statics also distinguish the two sides of recruiting: when employment expands, job finding becomes easier for workers while vacancy filling becomes harder for employers. Section 5 reports both probabilities.

The commuting-zone regression measures local comovement using an establishment-size-bin index averaged across industries. For the quantitative model, I use the employer-concentration distribution in [Berger, Herkenhoff, and Mongey \(2022\)](#), mapping each bin into a symmetric market with $HHI = 1/M$ to assign employer sizes and market weights. A direct comparison of regression coefficients would require the same market definition and aggregation of establishments by employer ownership.

D.3 Concentration proxy and sample

The outcome is 100 times LAUS unemployed persons divided by the labor force, aggregated from counties to commuting zones using the supplied crosswalk. Concentration is inferred from CBP establishment counts in employment-size bins. If c_{zjtb} is the establishment count in bin b of three-digit industry j and m_b its assigned employment midpoint, define

$$HHI_{zjt} = 10,000 \frac{\sum_b c_{zjtb} m_b^2}{(\sum_b c_{zjtb} m_b)^2}. \quad (\text{D.2})$$

This treats establishments within a bin as having the midpoint employment. A cell is retained only if all required counts are numeric, no required bin has a literal suppression flag, and the sum of bin counts agrees with the total number of establishments. Detailed bins above 1,000 workers use their separate midpoints when available; otherwise the aggregate 1,000+ bin uses 2,000. The commuting-zone measure is the equal-weighted mean across valid industry cells.

Establishments under common ownership are counted separately, so the resulting index measures establishment concentration.

³The job-finding calculations use coefficients -0.0284 and -0.0330 in Table 3, columns 8 and 6, multiplied by the HHI standard deviation 0.27 and by 100. Their clustered standard errors imply 0.14 and 0.06 percentage point for these contrasts.

The sample has 7,000 observations and 586 commuting zones. Outcome years are 2005–2016; lagged concentration years are 2004–2015. The labor-force-weighted mean unemployment rate is 6.65 percent and mean concentration is 1,295 on the 0–10,000 scale. After commuting-zone and year fixed effects, the weighted standard deviations are 0.790 percentage point for unemployment and 93 points for the level concentration proxy. This makes proportional concentration changes a more informative reporting unit than a 1,000-point within-zone change.

D.4 Estimation and figure construction

The preferred regression is (D.1), weighted by contemporaneous LAUS labor force. The coefficient on log HHI is 0.857, so a 10 percent increase in concentration is associated with $0.857 \log(1.1) \simeq 0.082$ percentage point higher unemployment. The standard error of this transformed effect is 0.036, and $p = 0.022$. The covariance matrix is clustered by commuting zone, with finite-sample multiplier

$$\frac{G}{G-1} \frac{n-1}{n-K},$$

where $G = 586$, $n = 7,000$, and $K = 599$ counts the commuting-zone and year categories plus the log-HHI coefficient. This convention includes all twelve year categories, although one year dummy is omitted from the fitted regression. The p -value uses the normal reference distribution.

For the graph, both variables are residualized using the same weighted fixed effects. Residualized log HHI is divided into 15 equal-observation quantile bins, with 466 or 467 observations per bin. Each plotted coordinate is a labor-force-weighted mean within the bin, and bubble area is proportional to the total labor force represented by that bin. The line is the full observation-level fixed-effects estimate through the residual origin.

D.5 Robustness and timing

Table 18 summarizes specification checks. The positive association persists with fixed labor-force weights and when the recession years 2008–2010 are omitted. It is smaller and less precise under equal observation weights, commuting-zone-specific trends, or tail trimming. The first-difference estimate is also positive and smaller than the preferred estimate.

Timing checks use concentration at alternative dates relative to unemployment. The estimated 10 percent effects are -0.0451 , 0.0154 , 0.0817 , 0.1421 , 0.1866 , 0.1798 , and 0.0923

Table 17: Preferred estimate and commuting-zone size

Sample or contrast	Effect (pp)	Clustered SE	<i>p</i> -value	Observations
Full sample	0.0817	0.0357	0.022	7,000
Small commuting zones	-0.0076	0.0546	0.890	3,484
Large commuting zones	0.0835	0.0365	0.022	3,516
Large minus small	0.0915	0.0627	0.144	7,000

Notes: Effects correspond to a 10 percent increase in lagged concentration. The preferred full-sample 95 percent interval is [0.0117, 0.1517] percentage point. Size is defined by the median of commuting zones' sample-period mean labor force, 74,514 workers. Each group has 293 zones. The last row reports the pooled-regression interaction contrast; equality between the groups has a *p*-value of 0.144.

Table 18: Robustness of the unemployment association

Specification	Effect (pp)	Clustered SE	<i>p</i> -value	Observations
Preferred weighted fixed effects	0.0817	0.0357	0.022	7,000
Fixed labor-force weights	0.0843	0.0363	0.020	7,000
Unweighted	0.0323	0.0297	0.276	7,000
Commuting-zone trends	0.0298	0.0443	0.502	7,000
Trim concentration tails	0.0358	0.0426	0.400	6,860
Omit 2008–2010	0.1032	0.0446	0.021	5,254
Balanced sample	0.0817	0.0357	0.022	6,984
First differences	0.0271	0.0157	0.085	6,414

Notes: Trimming excludes the first and 99th percentile concentration tails. Fixed weights use each zone's sample-period mean labor force. The balanced sample has 582 zones observed in all 12 years. First differences retain year effects and weight each transition by its two years' average labor force.

for dates $t-3, t-2, t-1, t, t+1, t+2, t+3$, respectively. Future concentration is also positively associated with current unemployment. These regressions describe their comovement over time, which may reflect both employer market power and persistent local demand conditions.

The U.S. panel measures unemployment stocks rather than worker transitions, so the job-finding evidence comes from [Berger et al. \(2024\)](#).

E Risk aversion, search, and policy derivatives

E.1 Profit income and proportional redistribution

The consumption equations describe the distribution of operating profits after wage payments and recruiting costs. Let r denote workers' share of equity income and let t_π be a fixed proportional levy on net operating profits, whose proceeds are rebated equally across the unit worker population. Workers then receive

$$r(1 - t_\pi)\pi + t_\pi\pi = [r + (1 - r)t_\pi]\pi = \phi\pi, \quad \phi = r + (1 - r)t_\pi. \quad (\text{E.1})$$

Outside recipients receive $(1 - r)(1 - t_\pi)\pi = (1 - \phi)\pi$. Hence the same ϕ can describe different combinations of ownership and redistribution. Profit receipts are the same for employed and unemployed workers, and an individual worker takes this common receipt as given in bargaining and search.

The profit-tax implementation uses a nonnegative time- and state-independent rate $t_\pi < 1$, applies to net operating profits with symmetric treatment of profits and losses, and rebates its proceeds as specified above. For given wage and policy functions, it multiplies the firm's discounted payoff by the positive constant $1 - t_\pi$, preserving its hiring choice. It also multiplies the firm's bargaining surplus by the same constant. The log Nash objective becomes

$$\chi \log S_{it} + (1 - \chi) \log [(1 - t_\pi)J_{it}] = \chi \log S_{it} + (1 - \chi) \log J_{it} + (1 - \chi) \log(1 - t_\pi). \quad (\text{E.2})$$

The last term is constant in the negotiation, so the bargaining condition is unchanged. The distribution of receipts still affects consumption, marginal utility, and the equilibrium wage through ϕ . The endpoint $\phi = 1$ is attained by full worker ownership and is also the limit of the rebate arrangement as t_π approaches one.

The UI exercises hold ϕ fixed and optimize b_0 using the common-worker financing rule. They therefore compare UI across profit-income arrangements, with the same household budgets and welfare criterion under either implementation.

E.2 Worker values and the search timing

Let E_{it} and U_t be worker values in utility units. Effort used at date t is $e_t = \mathcal{E}(\mathbf{n}_{t-1})$, chosen before the current target game. Define the contact rate per unit of effort as $a_t = p_t/e_t$. The worker recursions are

$$E_{it} = u_\sigma(c_{e,it}) + \beta[(1 - \delta)E_{i,t+1} + \delta U_{t+1}], \quad (\text{E.3})$$

$$U_t = u_\sigma(c_{u,t}) + \beta \max_{x \geq 0} \left\{ (1 - xa_{t+1})U_{t+1} + xa_{t+1} \sum_j \frac{h_{j,t+1}}{H_{t+1}} E_{j,t+1} - \Gamma(x) \right\}. \quad (\text{E.4})$$

The cost and benefit of search are both dated to the next matching interval. The first-order condition is (50); the discount factor cancels. Subtracting the two value equations gives

$$\begin{aligned} S_{it} &= u_\sigma(c_{e,it}) - u_\sigma(c_{u,t}) \\ &\quad + \beta \left[(1 - \delta)S_{i,t+1} - p_{t+1} \sum_j \frac{h_{j,t+1}}{H_{t+1}} S_{j,t+1} + \Gamma(e_{t+1}) \right]. \end{aligned} \quad (\text{E.5})$$

On a solution, $p\bar{S} = e\Gamma'(e)$, so the last two terms can be written as $\Gamma(e) - e\Gamma'(e) = -\psi\Gamma(e)$. The numerical system solves the surplus and effort equations separately and uses their reduced combination as an algebraic check.

The policy objective in (48) values consumption and effort costs in the stationary calendar-period allocation. Worker values in (E.3)–(E.4) instead determine private choices from a postmatching state, with search costs dated to the next matching interval.

E.3 Bargaining and the firm's envelope

In the marginal bilateral bargain, S is in utility units and J is in output units. The log Nash objective has first-order condition

$$\frac{\chi u'_\sigma(c_e)}{S} - \frac{1 - \chi}{J} = 0,$$

which yields (51). This condition is invariant to a positive rescaling of worker utility. At $\sigma = 0$ it reduces to the linear sharing rule. Common profit transfers and levies are fixed in the individual negotiation but endogenous across aggregate states.

Current effort is fixed in a target deviation, so the firm first-order condition remains (14), with the endogenous CRRA wage. For an inherited-state derivative, effort changes. Using

$q^{-1} \propto [H/(eu)]^{\nu/(1-\nu)}$, the envelope becomes

$$\begin{aligned} \frac{\partial \Pi_i(\mathbf{k})}{\partial k_k} &= (1-\delta) \frac{\kappa}{q} \mathbf{1}\{i=k\} \\ &\quad - h_i \frac{\nu}{1-\nu} \frac{\kappa}{q} \left[\frac{1}{u} - \frac{1-\delta}{H} - \frac{1}{\mathcal{E}(\mathbf{k})} \frac{\partial \mathcal{E}(\mathbf{k})}{\partial k_k} \right] \\ &\quad + \sum_{j \neq i} \frac{\partial \mathcal{N}_j(\mathbf{k})}{\partial k_k} \left[\beta \frac{\partial \Pi_i(\mathbf{n})}{\partial n_j} - n_i \frac{\partial w_i(\mathbf{n})}{\partial n_j} - h_i \frac{\nu}{1-\nu} \frac{\kappa}{qH} \right]. \end{aligned} \quad (\text{E.6})$$

The complete system consists of the firm condition, the worker surplus equation with non-linear bargaining substituted, the search first-order condition, and the cross part of (E.6). Own value derivatives are recovered from that same envelope.

At symmetry,

$$H = \delta N, \quad u = 1 - N, \quad V = \left(\frac{\delta N}{\mu[e(1-N)]^\nu} \right)^{1/(1-\nu)}, \quad p = \frac{\delta N}{1-N}.$$

Thus effort affects the vacancies needed to sustain a stationary N , whereas the stationary job-finding rate at a given N is pinned down by flows. Effort and employment are solved jointly from workers' search and firms' hiring conditions.

E.4 Deriving the stationary welfare decomposition

Write $L = 1 - N$. The differential of profits is

$$d\pi = \pi_N dN - N dw + \pi_e de, \quad \pi_N = \alpha A N^{\alpha-1} - w - \kappa V_N, \quad \pi_e = -\kappa V_e, \quad (\text{E.7})$$

where

$$V_N = \frac{V}{1-\nu} \left(\frac{1}{N} + \frac{\nu}{1-N} \right), \quad V_e = -\frac{\nu}{1-\nu} \frac{V}{e}. \quad (\text{E.8})$$

The common levy satisfies $dG = L db_0 - b_0 dN$. Hence

$$dc_e = dw + \phi d\pi - dG, \quad dc_u = db_0 + \phi d\pi - dG, \quad dc_K = 9(1-\phi)d\pi.$$

Differentiating (48) first gives

$$\begin{aligned} d\mathcal{W} &= [u_\sigma(c_e) - u_\sigma(c_u) + \Gamma(e)]dN + Nu'_\sigma(c_e)dw + Lu'_\sigma(c_u)db_0 \\ &\quad + \lambda d\pi - adG - L\Gamma'(e)de. \end{aligned} \quad (\text{E.9})$$

Substitute (E.7) and the levy differential, then group coefficients of db_0, dN, dw, de . Dividing by db_0 along the equilibrium response proves (53).

A wage increase contributes $Nu'_\sigma(c_e)w_b$ through labor income and $-N\lambda w_b$ through profits. When workers receive all profits, the two cancel if employed workers' marginal utility equals the average marginal value of worker income. When workers receive no profits, outside recipients' marginal utility determines the profit cost through λ .

E.5 Computing the equilibrium response

Let z collect the normalized coefficients and stationary levels in the complete numerical system $F(z, b_0) = 0$. Its implicit response is obtained from

$$F_z \frac{dz}{db_0} = -F_{b_0}. \quad (\text{E.10})$$

I use a parameter step 10^{-5} for the partial policy derivative of the residual and check the resulting responses against equilibria solved at $b_0 \pm 2 \times 10^{-4}$. The welfare derivative computed from those solutions is compared with the sum of the four components in (53).

I search over $b_0 \in [-0.39, 0.40]$ along each equilibrium branch, requiring positive worker consumption and feasible matching. Reported optima are interior and have small first-order residuals and negative local curvature.

E.6 Relation to the linear instrument representation

Under linear utility and common-worker financing, a common payment τ to firms per employed worker and UI enter the allocation through $\tau - b_0$. The allocation obtained at $(\tau, 0)$ is reproduced at $(0, -\tau)$, with the wage shifted by $-\tau$ so that the firm's effective labor cost is unchanged. Balanced-budget worker consumption also shifts consistently. The common-policy exercise in Appendix C.7 solves directly in b_0 , holding $\tau = 0$.

The CRRA exercise solves directly for UI under the profit-distribution and financing rules in Section 6. Appendix F studies employer-specific payments in asymmetric markets.

E.7 Calibration and approximation order

The CRRA calibration uses $\phi = 1$, the common-worker financing rule, and $b_0 = 0$. Let ℓ_g denote a market group’s labor-force weight. Mean effort among unemployed workers is normalized by

$$\frac{\sum_g \ell_g (1 - N_g) e_g}{\sum_g \ell_g (1 - N_g)} = 1. \quad (\text{E.11})$$

This fixes the units of effective search. Effort remains endogenous and differs across markets. Together with unemployment and the vacancy–unemployment ratio, the normalization determines matching efficiency, the recruiting cost, and the search-cost scale conditional on the specified preference parameters.

The fit uses the complete degree-one heterogeneous system and an exact atomistic endpoint. As in the linear calibration, market labor-force weights are inferred from empirical payroll shares and model payroll. The CRRA calibration’s weights are then held fixed when evaluating the higher-order approximation. Table 19 reports the resulting aggregate moments at the same primitives.

Table 19: Aggregate fit in the risk-averse economy

	Target	Degree one	Degree three
Unemployment (percent)	6.000	6.000	5.915
Vacancies / unemployment	0.720	0.720	0.733
Mean unemployed search effort	1.000	1.000	1.012

Notes: Full worker profit income, $b_0 = 0$, common-worker financing, and fixed CRRA calibration weights. Both approximations include markets with $M \in \{1, 2, 4, 10, 20\}$ and the exact atomistic endpoint. The recruiting cost, matching efficiency, and search-cost scale are unchanged across the two columns.

Moving to degree three lowers aggregate unemployment by 0.085 percentage point and raises the vacancy–unemployment ratio by 0.013. Mean effort among the unemployed rises by about 1.2 percent. Re-impling market weights from degree-three payroll instead gives unemployment of 5.913 percent, a vacancy–unemployment ratio of 0.734, and mean effort of 1.012. Thus changing the approximation order, rather than the small adjustment in implied market weights, accounts for most of the difference. The policy exercises retain the original fitted primitives throughout.

E.8 Sensitivity of optimal UI

Table 20 varies risk aversion, search-cost curvature, and the consumption of outside profit recipients. Each exercise retains the baseline recruiting cost, matching efficiency, search-cost scale, and other primitives, without refitting the aggregate targets.

The profit-income contrast persists across these variations. When workers receive all profits, optimal UI is lower under monopoly than at the atomistic endpoint. When outside profit recipients receive all profits, the ordering reverses. Greater risk aversion raises the optimal increment under monopoly for both profit-income arrangements. Changing search-cost curvature shifts the policy levels but preserves the contrast between the two market structures.

Table 20: Sensitivity of optimal UI to preferences and outside-recipient income

	Workers own all profits		Outside profit recipients receive all profits	
	Monopoly	Atomistic	Monopoly	Atomistic
Baseline	-0.2806	+0.0555	+0.2386	+0.1833
Risk aversion $\sigma = 0.25$	-0.3293	+0.0675	+0.1773	+0.1465
Risk aversion $\sigma = 0.75$	-0.2452	+0.0537	+0.2721	+0.2096
Search curvature $\psi = 0.4$	-0.3527	+0.0507	+0.2239	+0.1746
Search curvature $\psi = 0.8$	-0.2408	+0.0595	+0.2484	+0.1907
Outside-recipient endowment = 1	-0.2806	+0.0555	+0.2554	+0.1997
Outside-recipient mass = 2/9	-0.2806	+0.0555	+0.1902	+0.1379

Notes: Optimal increments b_0 , with baseline nonemployment income $b = 0.4$. The baseline has $\sigma = 0.5$, $\psi = 0.6$, outside-recipient mass 1/9 per worker, and no outside endowment. Each row changes only the stated assumption. Finite equilibria use degree three and the atomistic endpoint is solved directly. UI is financed by the common worker levy and the objective is stationary inclusive welfare. Outside-recipient mass and endowment leave the optimum at $\phi = 1$ unchanged because their utility is then constant across policies.

For the endowment exercise, outside-recipient consumption becomes $c_K = \bar{c}_K + (1 - \phi)\pi/m_K$, where $\bar{c}_K = 1$ is independent of employment and UI. This additional income lowers outside recipients' marginal utility and strengthens the welfare gain from transferring profit income to workers. Optimal UI consequently rises. Doubling outside-recipient mass to $m_K = 2/9$, while holding total profit income fixed, instead spreads those profits across more recipients. Their consumption falls and their marginal utility rises, reducing the preferred UI increment. Neither change enters the firm's or worker's equilibrium problem at a given policy; both change how the planner values its outcomes.

The policy search combines a grid with local refinement and checks the welfare derivative and curvature at the selected interior optimum. The interval is $[-0.39, 0.40]$, except for $\sigma = 0.75$ with no worker profit income, where it is $[-0.30, 0.40]$: continuation to the lowest

original policy did not produce an accepted equilibrium. All reported optima have negative local curvature and lie well inside their search intervals.

E.9 Welfare contributions across concentration

Table 21 reports the contributions behind Figure 12 for the two endpoint profit-income arrangements.

Table 21: Marginal UI welfare contributions at a common zero policy

Profit share	M	Insurance	Employment	Wage/rents	Search	Total
100%	1	+0.0110	-0.0868	-0.0067	+0.0424	-0.0401
0%	1	+0.0250	-0.1339	+0.5061	+0.0948	+0.4920
100%	3	+0.0101	-0.0312	-0.0017	+0.0258	+0.0031
0%	3	+0.0195	-0.1143	+0.1183	+0.0884	+0.1119
100%	10	+0.0095	-0.0030	-0.0011	+0.0080	+0.0135
0%	10	+0.0175	-0.0832	+0.0714	+0.0667	+0.0723
100%	∞	+0.0089	+0.0066	-0.0010	-0.0009	+0.0136
0%	∞	+0.0168	-0.0820	+0.0696	+0.0644	+0.0687

Notes: The first column is the proportion of profits received by workers. All derivatives are evaluated at $b_0 = 0$ under common-worker financing. Employment includes output and financing; search includes recruiting costs. The unrounded columns add to the full equilibrium derivative. The maximum discrepancy against a derivative obtained by resolving neighboring policy equilibria is 1.44×10^{-7} .

E.10 Employment responses and the welfare value of jobs

The employment contribution in (53) is the product of the equilibrium response N_b and the partial welfare value of employment. Writing stationary welfare as a function of N , w , e , and b_0 , define

$$\begin{aligned} \mathcal{W}_N &\equiv \left. \frac{\partial \mathcal{W}}{\partial N} \right|_{w,e,b_0} \\ &= u_\sigma(c_e) - u_\sigma(c_u) + \Gamma(e) + \lambda\pi_N + b_0 a. \end{aligned} \tag{E.12}$$

Consumption, profits, and the financing levy adjust with employment according to (44)–(46). Wages, search effort, the UI increment, and the profit-income arrangement are held fixed in this partial derivative. The employment contribution is therefore $\mathcal{W}_N N_b$.

Table 22 reports both factors at the common policy $b_0 = 0$. At $\phi = 1$, $\mathcal{W}_N$ falls from 0.2763 under monopoly to -0.0342 at the atomistic endpoint, while N_b remains negative. The sign change in the employment contribution consequently reflects the welfare value of

employment. At $\phi = 0$, the endpoint values of $\mathcal{W}_N$ are nearly identical, and the difference in employment costs instead reflects the magnitude of N_b . At intermediate profit shares, both factors contribute to the comparison.

Table 22: Employment responses and the welfare value of jobs at zero UI

Worker profit share	M	Employment response N_b	Welfare value $\mathcal{W}_N$	Employment contribution
100%	1	-0.3143	+0.2763	-0.0868
100%	3	-0.2261	+0.1381	-0.0312
100%	10	-0.1832	+0.0161	-0.0030
100%	∞	-0.1920	-0.0342	+0.0066
50%	1	-0.3135	+0.3294	-0.1033
50%	3	-0.2243	+0.2279	-0.0511
50%	10	-0.1809	+0.1086	-0.0197
50%	∞	-0.1926	+0.0635	-0.0122
10%	1	-0.3152	+0.4017	-0.1266
10%	3	-0.2237	+0.4433	-0.0991
10%	10	-0.1796	+0.3793	-0.0681
10%	∞	-0.1934	+0.3378	-0.0653
0%	1	-0.3164	+0.4233	-0.1339
0%	3	-0.2238	+0.5109	-0.1143
0%	10	-0.1794	+0.4640	-0.0832
0%	∞	-0.1937	+0.4235	-0.0820

Notes: All rows use $b_0 = 0$, common-worker financing, and the fixed CRRA calibration. Finite markets use the degree-three equilibria underlying Figure 12; the atomistic endpoint is solved from its limiting equations. N_b is a local derivative of the employment share with respect to the UI increment, not an elasticity. $\mathcal{W}_N$ is measured in stationary welfare units per unit of the employment share. The final column is the product of the preceding two columns before rounding.

The welfare values are reconstructed from consumption and matching primitives. With search effort fixed,

$$V_N = \frac{V}{1 - \nu} \left(\frac{1}{N} + \frac{\nu}{1 - N} \right), \quad \pi_N = \alpha A N^{\alpha-1} - w - \kappa V_N. \quad (\text{E.13})$$

At $b_0 = 0$, the financing component $b_0 a$ is zero. The reconstructed products reproduce the saved employment contributions to within 2×10^{-16} across the zero-policy and optimal-policy records. An independent central-difference calculation of stationary welfare at fixed w , e , and b_0 agrees with (E.12) to within 2.5×10^{-10} . The saved employment derivatives also agree with derivatives obtained from neighboring policy equilibria to within 3.4×10^{-7} .

Production and recruiting behind the value of employment. At the common policy $b_0 = 0$, the partial welfare value of employment is

$$\mathcal{W}_N = u_\sigma(c_e) - u_\sigma(c_u) + \Gamma(e) + \lambda [\alpha AN^{\alpha-1} - w - \kappa V_N]. \quad (\text{E.14})$$

Wages and search effort are held fixed in this partial derivative. As shown in (E.13), an additional job requires replacement hiring and reduces the pool available to fill vacancies. Both enter the recruiting cost of sustaining higher employment.

Table 23: The welfare value of employment when workers receive all profits

	Monopoly	Atomistic
Consumption gain, $u_\sigma(c_e) - u_\sigma(c_u)$	+0.1723	+0.2755
Avoided search cost, $\Gamma(e)$	+0.1942	+0.3846
Output contribution, $\lambda \alpha AN^{\alpha-1}$	+0.6923	+0.6713
Wage cost through profits, $-\lambda w$	-0.5784	-0.6693
Recruiting cost, $-\lambda \kappa V_N$	-0.2042	-0.6962
Welfare value of employment, $\mathcal{W}_N$	+0.2763	-0.0342

Notes: Common policy $b_0 = 0$, $\phi = 1$, baseline CRRA primitives. The monopoly allocation uses degree three and the atomistic allocation is solved directly. Entries are the components of (E.14) evaluated at each equilibrium, not counterfactuals that hold the other components fixed.

The output contribution changes relatively little across the two allocations. The larger marginal recruiting cost accounts for the largest negative change in the value of employment, partly offset by a larger consumption gain and greater search costs avoided when a worker becomes employed. The distribution of profit income changes this comparison through both equilibrium consumption and the welfare value of profits. With no worker profit income, $\mathcal{W}_N$ is approximately 0.423 at both endpoints. The comparison at $\phi = 1$ therefore differs from the case with $\phi = 0$, where the employment response accounts for almost all of the difference in the employment contribution.

E.11 UI contributions along the policy path

The common-policy decomposition compares market structures at the same benefit level. To explain the preferred policies, I also trace the contributions as UI moves from zero to each optimum. All primitives, worker profit shares, and the financing rule remain fixed along a path. Each point is a stationary equilibrium. The exercise compares monopoly with the independently solved atomistic economy, at $\phi = 1$ and $\phi = 0$.

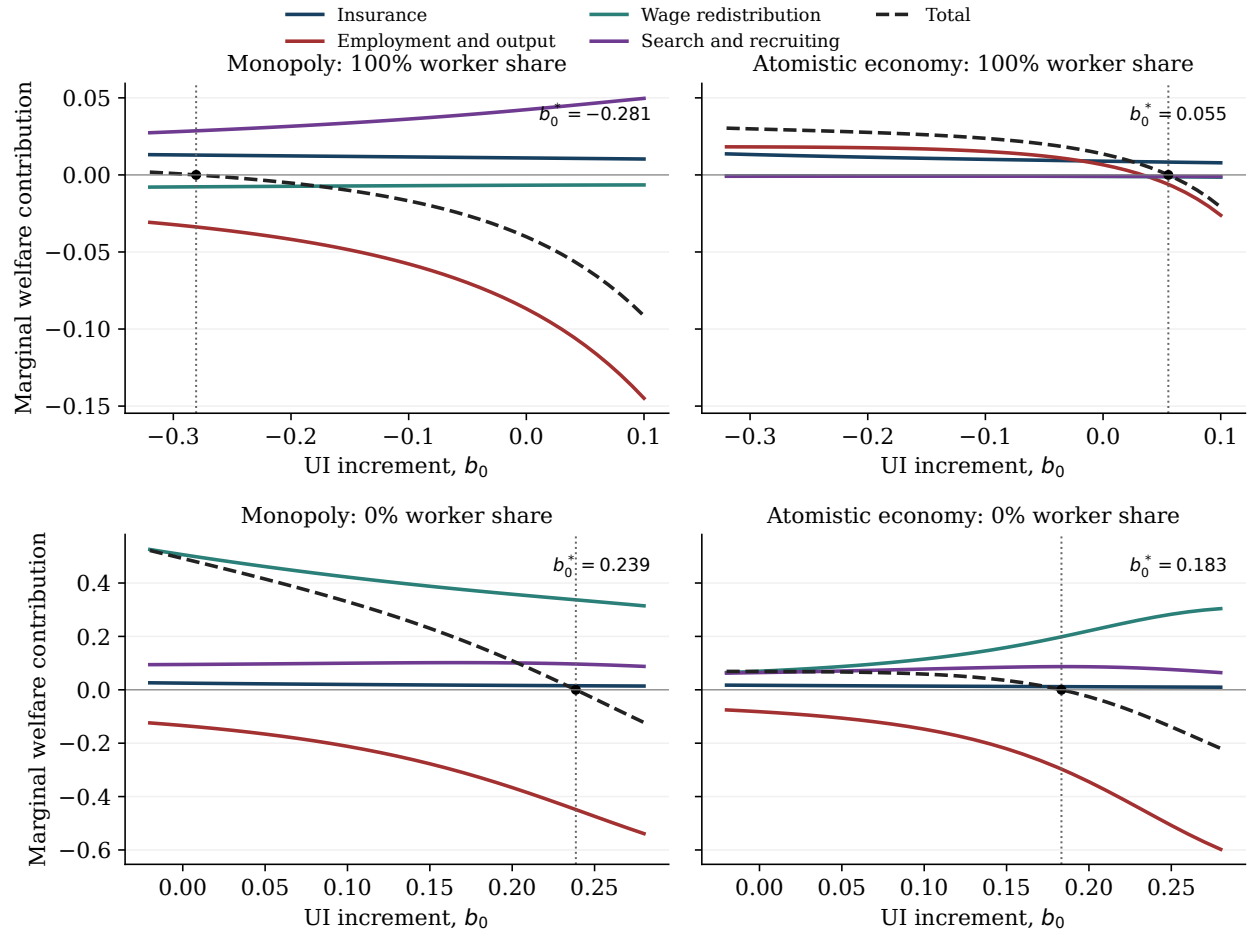


Figure 15: Marginal welfare contributions along the UI policy path

Notes: Each point is a separately solved stationary equilibrium. The dashed black line is the sum of the four contributions. The vertical dotted line and black dot mark the optimum. Vertical scales are common within each profit-share row.

Write the four contributions in (53) as insurance $I(b_0)$, employment $E(b_0)$, wage redistribution $R(b_0)$, and search $Q(b_0)$. The employment term is the product of N_b and the job value defined in (E.12). The wage term is the product of Nw_b and the redistribution value $u'_\sigma(c_e) - \lambda$.

Figure 15 plots the four contributions and their sum. Table 24 separates the employment contribution into its response and welfare value, and does the same for wage redistribution. When workers receive all profits, the negative optimal monopoly increment raises employment. Moving toward that policy reduces the absolute employment response from 0.314 to 0.111, while the value of employment rises from 0.276 to 0.305. The marginal employment benefit of further reducing UI therefore declines. When workers receive no profits, a positive monopoly increment instead increases the absolute employment response from 0.316 to 1.068; the value of employment is approximately unchanged. This growing employment cost limits the benefit increase.

The relative employment responses also change with the common policy level. When workers receive no profits, the atomistic response becomes larger in absolute value at a common increment of approximately 0.17, as shown in Figure 17. The ordering at zero UI changes as the policy moves the two economies to different employment and search allocations.

At the common zero policy with $\phi = 0$, the redistribution value is 0.842 under monopoly and 0.660 in the atomistic economy. The corresponding wage responses are 0.688 and 0.111. The much larger wage response is thus central to the monopoly redistribution benefit. At the respective optima, the redistribution values are almost equal, whereas the monopoly wage response remains about twice as large.

Table 24: Equilibrium responses and welfare values at zero and optimal UI

Profit share	Market	Policy	N_b	$\mathcal{W}_N$	w_b	$u'_\sigma(c_e) - \lambda$
100%	Monopoly	Zero	-0.314	0.276	0.607	-0.013
100%	Monopoly	Optimal	-0.111	0.305	0.600	-0.014
100%	Atomistic	Zero	-0.192	-0.034	0.112	-0.009
100%	Atomistic	Optimal	-0.279	0.022	0.147	-0.009
0%	Monopoly	Zero	-0.316	0.423	0.688	0.842
0%	Monopoly	Optimal	-1.068	0.420	0.747	0.621
0%	Atomistic	Zero	-0.194	0.424	0.111	0.660
0%	Atomistic	Optimal	-0.861	0.345	0.366	0.624

Notes: The first column gives the worker profit share; 100 percent and zero percent denote the two endpoint arrangements. The employment contribution is $\mathcal{W}_N N_b$ and the wage contribution is $N[u'_\sigma(c_e) - \lambda]w_b$. The optimal increments are -0.281 and 0.055 when workers receive all profits, and 0.239 and 0.183 when workers receive no profits, for monopoly and the atomistic economy respectively.

Accounting for the total welfare gain. Integrating the four derivatives along the policy path gives

$$\mathcal{W}(b_0^*) - \mathcal{W}(0) = \int_0^{b_0^*} I(b_0) db_0 + \int_0^{b_0^*} E(b_0) db_0 + \int_0^{b_0^*} R(b_0) db_0 + \int_0^{b_0^*} Q(b_0) db_0. \quad (\text{E.15})$$

When workers receive all profits, moving to the negative monopoly optimum lowers nonemployment income and raises employment. Integration runs from zero toward lower benefits, giving the positive employment contribution in Table 25. This employment gain exceeds the total welfare gain because lower nonemployment income also reduces insurance and increases search costs net of recruiting savings. When workers receive no profits, the largest positive contribution is wage redistribution; employment accounts for the principal offset.

Table 25: Contributions to the welfare gain from optimal UI

Profit share	Market	Insurance	Employment	Wages	Search	Total
100%	Monopoly	-0.0033	+0.0150	+0.0020	-0.0097	+0.0039
100%	Atomistic	+0.0005	+0.0001	-0.0001	-0.0001	+0.0004
0%	Monopoly	+0.0047	-0.0607	+0.0985	+0.0236	+0.0662
0%	Atomistic	+0.0026	-0.0285	+0.0216	+0.0140	+0.0097

Notes: Integrals from zero UI to the respective optimum, in stationary welfare units per worker. Entries are changes in utility levels. Each contribution is evaluated along the same equilibrium policy path; columns add before rounding.

E.12 Bargaining, search, and the wage response to UI

To decompose the wage response, let S be the worker's employment surplus in utility units and define $d_0 = 1 - \beta(1 - \delta)$ and $K_M = 1 + \nu / [(1 - \nu)M]$, with $K_\infty = 1$. The firm's first-order condition and the sharing rule imply

$$S = \frac{\chi}{1 - \chi} u'_\sigma(c_e) \frac{\kappa K_M}{q}. \quad (\text{E.16})$$

The stationary worker recursion and search condition are

$$u_\sigma(c_e) - u_\sigma(c_u) = [d_0 + \beta p]S - \beta \Gamma(e), \quad pS = e \Gamma'(e) = (1 + \psi) \Gamma(e).$$

Combining them gives

$$u_\sigma(c_e) - u_\sigma(c_u) = [1 - \beta(1 - \delta)]S + \beta \psi \Gamma(e). \quad (\text{E.17})$$

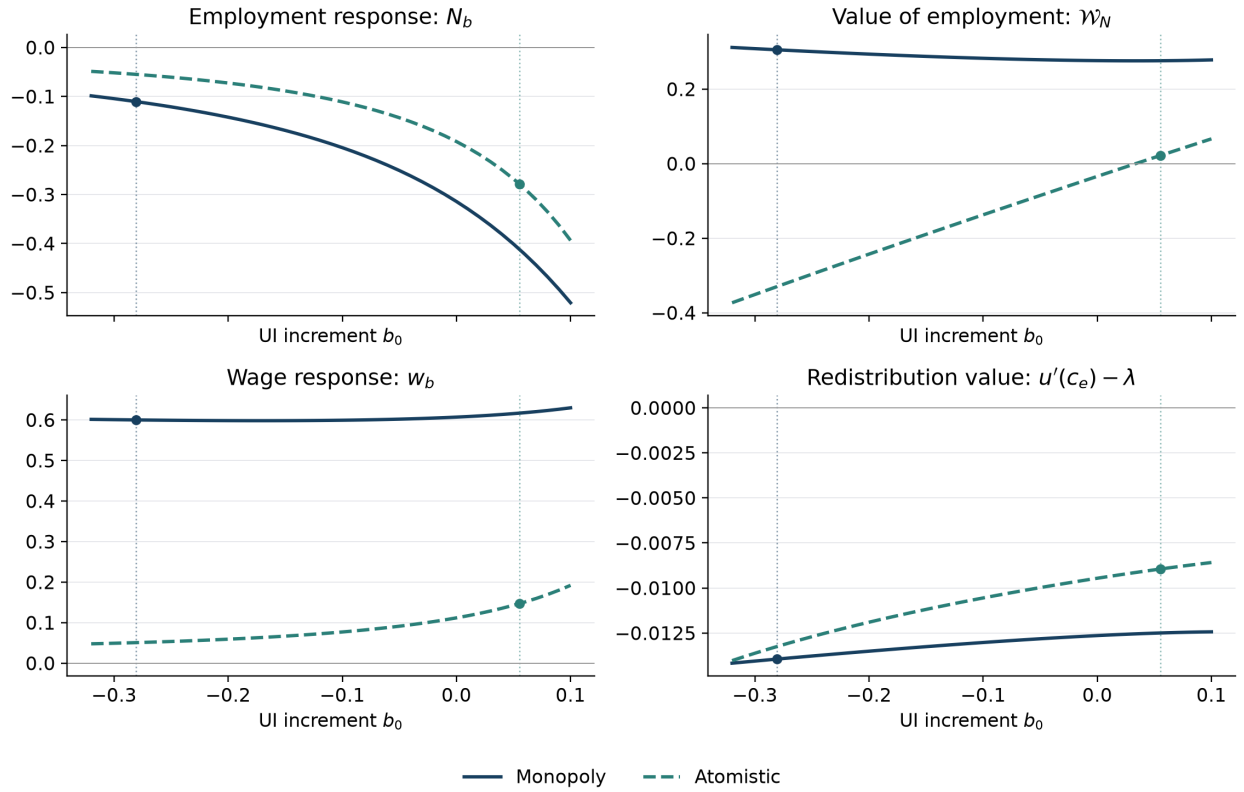


Figure 16: Employment and wage factors: $\phi = 1$

Notes: Dots and matching vertical lines identify each market's optimum. The wage contribution also includes the employment share N . The value of employment is the partial stationary welfare derivative, holding the wage and effort fixed.

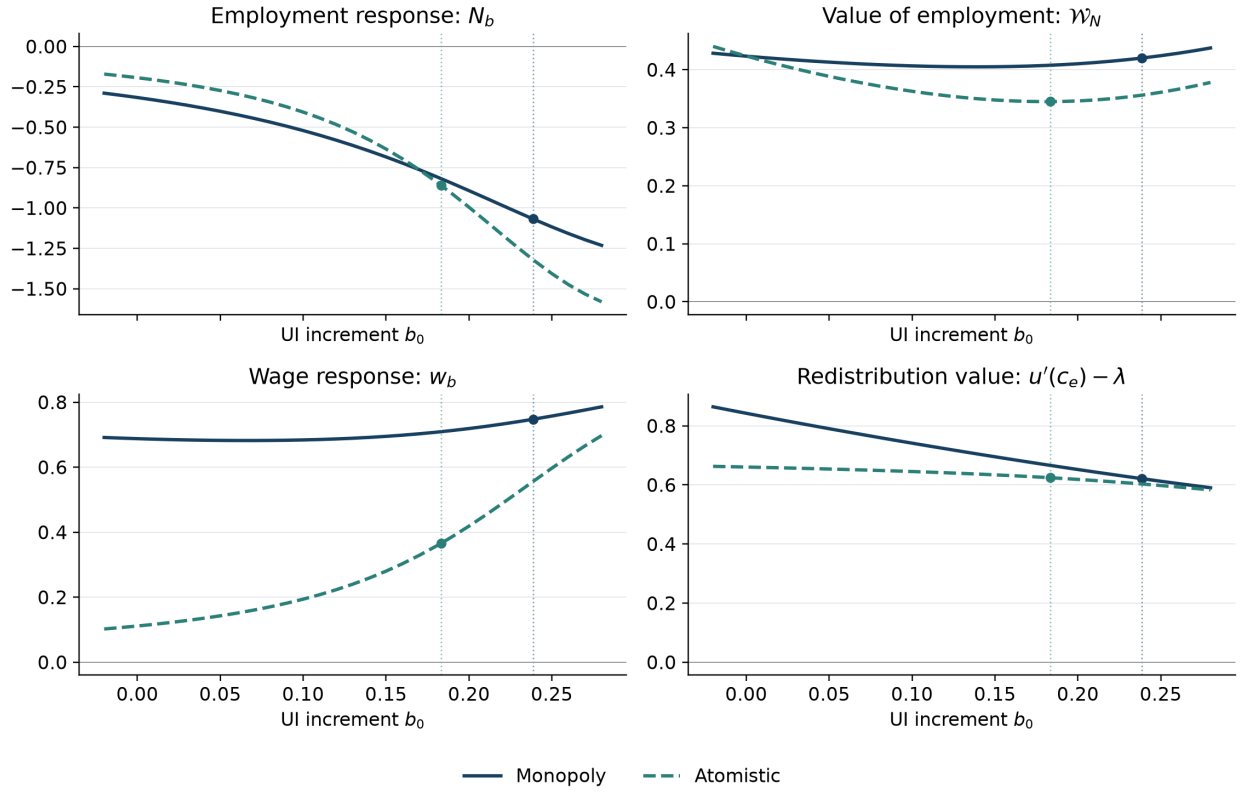


Figure 17: Employment and wage factors: $\phi = 0$

Notes: Definitions and markers are the same as in Figure 16. In monopoly, the employment response becomes substantially more negative as UI rises, while the welfare value of employment changes relatively little.

This identity connects the wage bargain to the utility advantage needed to sustain search. At unchanged employment, effort, and profit transfers, higher UI narrows the employed–unemployed consumption gap and puts upward pressure on the negotiated wage. A reduction in search effort lowers the right-hand side and offsets that pressure.

An exact derivative accounting. Write $u'_e = u'_\sigma(c_e)$ and $u'_u = u'_\sigma(c_u)$. Define

$$T = u'_e + \frac{d_0 \sigma S}{c_e}, \quad D_w = T(1 - \phi N) + \phi N u'_u.$$

Because $0 \leq \phi \leq 1$ and $N < 1$, $D_w > 0$. The consumption derivatives are

$$\begin{aligned} c_{e,b} &= (1 - \phi N)w_b + \phi(\pi_N N_b + \pi_e e_b) + b_0 N_b - (1 - N), \\ c_{u,b} &= -\phi N w_b + \phi(\pi_N N_b + \pi_e e_b) + b_0 N_b + N. \end{aligned}$$

Differentiate (E.17), use $S_b/S = -\sigma c_{e,b}/c_e - q_b/q$, and collect w_b terms. This yields

$$\begin{aligned} w_b = & \underbrace{\frac{(1 - N)T + N u'_u}{D_w}}_{\text{direct UI}} - \underbrace{\frac{(T - u'_u)\phi(\pi_N N_b + \pi_e e_b)}{D_w}}_{\text{profit income}} \\ & - \underbrace{\frac{(T - u'_u)b_0 N_b}{D_w}}_{\text{financing}} - \underbrace{\frac{d_0 S}{D_w} \frac{q_N N_b + q_e e_b}{q}}_{\text{vacancy filling}} + \underbrace{\frac{\beta \psi \Gamma'(e)}{D_w} e_b}_{\text{search adjustment}}. \end{aligned} \quad (\text{E.18})$$

Here $q_N/q = -\nu/[(1 - \nu)N(1 - N)]$ and $q_e/q = \nu/[(1 - \nu)e]$. The wage-induced change in profit transfers, $-\phi N w_b$, is included in D_w ; the profit-transfer term displayed separately contains only the employment and effort responses. Financing captures the change in the common levy caused by a change in employment, after the direct cost of a benefit increase has been included in the first term.

Table 26: Accounting for the wage response at the common zero-UI policy

Profit share	Market	Direct UI	Search	Filling	Income	w_b
100%	Monopoly	+1.0000	-0.3590	-0.0332	-0.0009	+0.6070
100%	Atomistic	+1.0000	-0.8609	-0.0316	+0.0043	+0.1118
0%	Monopoly	+1.1017	-0.3920	-0.0220	+0.0000	+0.6877
0%	Atomistic	+1.2167	-1.0703	-0.0352	+0.0000	+0.1112

Notes: Signed contributions in (E.18); Income combines profit transfers and financing. The first four numerical columns sum to w_b , up to rounding. All terms follow the jointly solved equilibrium response.

Table 26 shows that the search adjustment accounts for most of the wage-response difference between the two market structures. When workers receive all profits, the direct contribution is one in both markets, while search offsets 0.359 under monopoly and 0.861 in the atomistic economy. The remaining contributions are small in net terms. When workers receive no profits, the direct contribution is actually larger in the atomistic economy, but the search offset is also much larger: 1.070, compared with 0.392 under monopoly. The resulting wage response is 0.111 rather than 0.688. Figure 18 follows the same accounting as benefits change.

Why the search response differs. Using the matching function and (E.16) in the search condition gives the derivative identity

$$\left(\psi + \frac{1}{1-\nu}\right) \frac{e_b}{e} = \frac{N_b}{(1-\nu)N(1-N)} - \sigma \frac{c_{e,b}}{c_e}. \quad (\text{E.19})$$

In stationary equilibrium, job finding satisfies $p = \delta N/(1-N)$. When unemployment is low, a given change in employment produces a larger proportional change in job finding. Together with the filling-rate response, this creates the factor $1/[N(1-N)]$ in (E.19). Higher baseline effort also magnifies the change in effort levels. At zero UI with $\phi = 1$, the employment-and-matching term contributes -1.421 to e_b under monopoly and -2.791 in the atomistic economy; the consumption terms are only 0.009 and 0.003. When workers receive no profits the corresponding matching terms are -1.674 and -3.471 , with consumption terms -0.150 and -0.020 . Thus the stronger atomistic effort response arises mainly from the response of search incentives through employment and matching conditions.

Equations (E.18) and (E.19) describe the joint adjustment of employment, wages, and effort along each policy path. They assign the wage response to terms in the bargaining and search conditions evaluated at the same equilibrium. The endpoint comparison concerns own recruiting and search responses; rival-policy feedback operates in the intermediate oligopsony markets.

Numerical verification. The exercise uses 344 policy evaluations across the four cases. Monopoly responses come from implicit differentiation of the complete degree-three equilibrium system and are checked against separately solved neighboring policies. Atomistic responses use centered differences of the independently solved endpoint. The policy step is 5×10^{-5} , with half-step checks at zero and optimal UI. The largest equilibrium residual is below 10^{-8} . Across the paths, the welfare derivative agrees with independently differ-

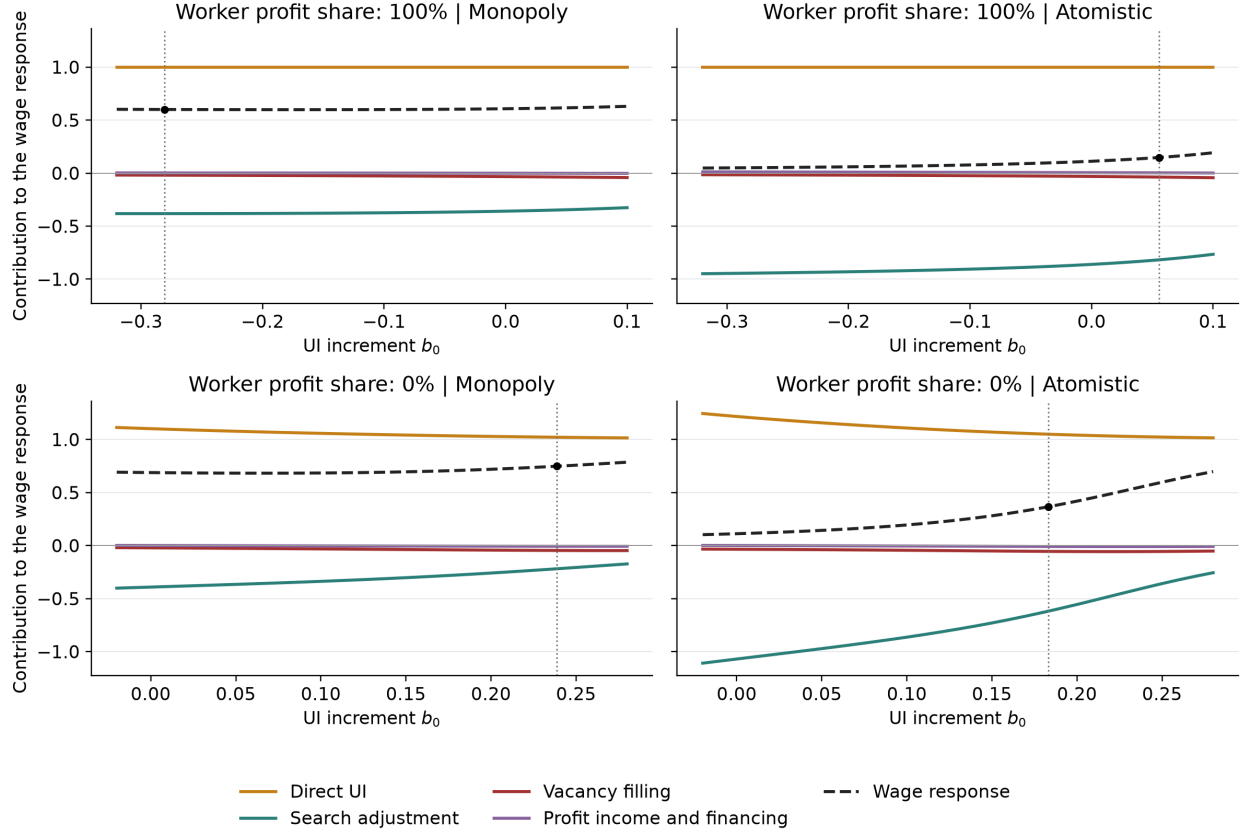


Figure 18: Bargaining and search contributions to the wage response

Notes: Contributions from (E.18). The dashed black line is their sum, w_b . Profit transfers and financing are combined; the vacancy-filling term includes the employment and effort responses. Vertical lines identify the respective optimal policies. All panels share the same vertical scale.

enced welfare to 1.3×10^{-8} ; the wage and search derivative identities close to 2.5×10^{-8} and 1.7×10^{-7} , respectively. Differentiating the unreduced worker recursion provides an independent wage-accounting check, with maximum discrepancy 1.5×10^{-7} . Sixteen- and 32-node Gauss–Legendre quadrature agree within 1.2×10^{-11} for every integrated contribution. Their sum reproduces the directly calculated welfare gain within 4.6×10^{-10} .

F Supplementary market-structure and productivity exercises

This appendix studies mergers, employer-specific remedies, and productivity changes.

F.1 Mergers with unchanged productive capacity

With asymmetric employers, write production as $A_i n_i^\alpha$. The aggregate capacity index is $\sum_i A_i^{1/(1-\alpha)}$. A zero-synergy merger of two employers preserves the sum of their capacities:

$$A_D^{1/(1-\alpha)} = A_1^{1/(1-\alpha)} + A_2^{1/(1-\alpha)}.$$

This keeps the efficient aggregate production frontier unchanged but combines two strategic decision makers into one. A merger of two employers in an initially symmetric four-employer market yields one dominant firm and two exchangeable fringe firms. The post-merger system has separate dominant and fringe policies and includes all symmetry-allowed derivatives within the fringe.

Without a remedy, the degree-five post-merger equilibrium has employment 0.928432 and payroll HHI 3616.10. The planner assigns employment 0.465650 to the merged firm and 0.232825 to each fringe employer.

F.2 Aggregate versus employer-specific remedies

Consider payments per employed worker of τ_D to the dominant firm and τ_F to each fringe employer. These instruments can target the distribution of employment as well as its aggregate level.

Table 27: Remedies following the zero-synergy 4 → 3 merger

Rule	τ_D	τ_F	$n_D - n_D^P$	Welfare gap
Employer-specific vector	0.055346	0.011935	0.000000	0.000000
Common incentive	0.031597	0.031597	-0.031765	-0.000513
Dominant firm only	0.066419	0.000000	0.018300	-0.000170
Fringe firms only	0.000000	0.052490	-0.067434	-0.002321

Notes: Every scalar rule matches aggregate planner employment. The final column is stationary aggregate welfare minus the dynamic planner's stationary welfare. Fiscal costs vary across rules. All solutions use the degree-five dominant–fringe system.

The common instrument restores total employment but leaves too few workers at the dominant firm and too many at the fringe relative to the planner. These employment differences offset in the aggregate, but the allocation produces less output. Among the scalar rules in Table 27, targeting the dominant firm comes closest to the planner’s welfare.

Employer-specific payments implement both employment targets, raising output by re-allocating jobs at the same total employment.

F.3 Merger and breakup thresholds

Zero-synergy mergers increase unemployment by 1.453, 0.605, and 0.329 percentage points for $2 \rightarrow 1$, $3 \rightarrow 2$, and $4 \rightarrow 3$, respectively. The associated changes in stationary aggregate welfare are -0.001320 , -0.000786 , and -0.000466 . The asymmetric cases use the dominant–fringe model. For the $4 \rightarrow 3$ merger, solving the equilibrium at different productivity levels gives a welfare break-even productivity gain of about 0.098 percent.

The reverse symmetric breakup exercise compares a monopoly with $K = 2, 3, 4, 5$ employers, allowing a common loss in productive capacity. The stationary-welfare break-even productivity factors are 0.998617, 0.998390, 0.998353, and 0.998358. Thus productivity losses of roughly 0.14–0.17 percent exhaust the aggregate efficiency gain from breakup. Workers’ gains also depend on how profit income is distributed.

F.4 Productivity and propagation

The permanent-productivity exercise compares stationary equilibria after a common change in A . The reported employment elasticity is 0.0931 under monopoly and 0.0578 in the atomistic economy. The wage elasticity rises from 0.773 to 0.977. Concentrated employers therefore allocate more of a permanent productivity increase to employment and less to wages in this calibration. Tightness elasticities differ much less: approximately 2.048 under monopoly, around 2.25 at ten and twenty employers, and 2.096 at the atomistic endpoint.

A local stochastic extension adds a persistent common-productivity state. Tightness amplification is about 2 percent greater than in the atomistic benchmark, leaving most of the unemployment–vacancy volatility puzzle emphasized by [Shimer \(2005\)](#) intact.

Concentration has larger effects on stationary employment and distribution than on these local productivity responses.

G Employer size and a competitive fringe

The derivations and additional comparisons below use the fringe economy in Sections 4.4 and 5.6. All numerical exercises retain the linear-utility baseline and zero-policy calibration.

G.1 Capacity, hiring shares, and recruiting incentives

Production is given by (33). The atomistic fringe is constructed as the limit of F employers, each with capacity c_f/F , as F tends to infinity. A fringe employer has no individual effect on matching conditions in this limit. Its within-firm bargaining effect remains finite when expressed as employment times the own wage slope.

The state and simultaneous-choice timing are as in Section 4.4. The large-employer problem includes aggregate market impact; the private fringe problem takes aggregate conditions as given.

For the detailed derivations, write $\eta = \nu/(1 - \nu)$, $d = 1 - \beta(1 - \delta)$, and $D = d + \beta p$. The signed recruiting contribution corresponding to (35) is

$$T_i = -s_i \frac{\eta \kappa}{q} [d + \beta(1 - \chi)p] - \beta \chi \eta \kappa \theta \mathcal{H}.$$

The own-share and concentration terms reported below sum to this contribution. In the no-fringe economy, $s_i = \mathcal{H} = 1/M$, recovering the recruiting block in (19).

G.2 Bargaining and the complete hiring condition

For a large employer, the Bellman problem is

$$\Pi_i(\mathbf{k}) = \max_{n_i} \left\{ Y_i(n_i) - n_i w_i(\mathbf{n}) - \frac{\kappa[n_i - (1 - \delta)k_i]}{q(\mathbf{k}, \mathbf{n})} + \beta \Pi_i(\mathbf{n}) \right\}, \quad (\text{G.1})$$

where other employers' current targets are fixed at their Markov policies. An individual fringe employer solves the corresponding private problem holding aggregate matching conditions and aggregate future policies fixed. The aggregate fringe policy is obtained by aggregating those private choices. Thus derivatives of aggregate fringe hiring do not enter an atom's unilateral choice, although they enter a large employer's continuation value.

Let $\mathcal{S}_g = E_g - U$ denote the employment surplus at employer type g . The worker recursion and bargaining condition are

$$\mathcal{S}_g = w_g - b - b_0 + \beta[(1 - \delta)\mathcal{S}'_g - p'\bar{\mathcal{S}}'], \quad \bar{\mathcal{S}} = \sum_i s_i \mathcal{S}_i + s_f \mathcal{S}_f, \quad (\text{G.2})$$

$$(1 - \chi)\mathcal{S}_i = \chi J_i, \quad J_i = MP_i - w_i - n_i w_{i,n_i} + \beta \Pi_{i,n_i}. \quad (\text{G.3})$$

Here primes refer to the next hiring interval. The hiring condition sets $J_i = \kappa(1 + \eta s_i)/q$, while a fringe atom has $J_f = \kappa/q$. Taking the destination-weighted average proves (34). A fringe consisting of F equal atoms contributes s_f^2/F to the sum of squared shares, which tends to zero.

At a stationary allocation, the full large-employer wage equation is

$$\begin{aligned} w_i = & (1 - \chi)(b + b_0) + \chi(MP_i - n_i w_{i,n_i}) + \beta \chi \kappa \theta [1 + \eta(\mathcal{H} - s_i)] \\ & + \beta \chi \sum_{g \neq i} \mathcal{N}_{g,n_i} \left[\beta \Pi_{i,n_g} - n_i w_{i,n_g} - s_i \frac{\eta \kappa}{q} \right]. \end{aligned} \quad (\text{G.4})$$

The index g includes the other large employers and the aggregate fringe. Policy derivatives describe future responses to the employment state. The regular fringe wage, with $x_f = n_f/c_f$, is

$$w_f = (1 - \chi)(b + b_0) + \frac{\chi \alpha A}{1 - \chi(1 - \alpha)} x_f^{\alpha-1} + \beta \chi \kappa \theta (1 + \eta \mathcal{H}). \quad (\text{G.5})$$

It retains the ordinary within-firm wage-dilution component and the common worker outside option.

For the hiring equation, define the remaining signed Euler contributions by

$$G_i = -(1 - \chi)n_i [w_{i,n_i} - w_{i,n_i}^{\text{SZ}}], \quad K_g = \beta \Pi_{i,n_g} - n_i w_{i,n_g} - s_i \eta \kappa / q, \quad (\text{G.6})$$

$$R_i = \beta(1 - \chi) \sum_{j \neq i, j \leq M} \mathcal{N}_{j,n_i} K_j, \quad F_i = \beta(1 - \chi) \mathcal{N}_{f,n_i} K_f. \quad (\text{G.7})$$

The hiring Euler equation is

$$\begin{aligned} 0 = & (1 - \chi) [MP_i - b - b_0 - n_i w_{i,n_i}^{\text{SZ}}] - d\kappa/q - \beta \chi \kappa \theta \\ & + T_i + G_i + R_i + F_i. \end{aligned} \quad (\text{G.8})$$

The terms preceding T_i form the canonical multiworker benchmark. The remaining terms separate recruiting, the GE wage-bill effect, large-rival policies, and aggregate fringe ad-

justment. Large-rival responses also enter the wage equation, so R_i measures the explicit rival-policy term in the Euler equation.

Proof of Proposition 3. Before substituting bargaining, the own recruiting effect in signed Euler units is $-s_i\eta\kappa D/q$. Substituting (G.4) adds the recruiting-related bargaining term $\beta\chi\eta\kappa\theta(s_i - \mathcal{H})$. Since $p = q\theta$, their sum equals T_i in (35). For a fringe atom, own share is zero, so $T_f = -\beta\chi\eta\kappa p\mathcal{H}/q$. The hiring-weighted sum of the bargaining corrections is proportional to

$$\sum_i s_i(s_i - \mathcal{H}) + s_f(0 - \mathcal{H}) = \mathcal{H} - \mathcal{H} \left(\sum_i s_i + s_f \right) = 0.$$

The remaining own-exposure term is proportional to $\sum_i s_i^2 = \mathcal{H}$, proving (36).

Recursive future responses. Index cross states by the other large employers and aggregate fringe employment. Let $a_g = \mathcal{N}_{g,n_i}$ and $Q_{ga} = \mathcal{N}_{g,n_a}$ for $g, a \neq i$. Cross envelopes yield

$$(I - \beta Q^\top)K = -n_i w_{i,n_{-i}} - s_i \frac{\eta\kappa D}{q} \mathbf{1}. \quad (\text{G.9})$$

The combined future-policy contribution is

$$R_i + F_i = -\beta(1 - \chi)a^\top (I - \beta Q^\top)^{-1} \left[n_i w_{i,n_{-i}} + s_i \frac{\eta\kappa D}{q} \mathbf{1} \right]. \quad (\text{G.10})$$

When the discounted cross-policy matrix has spectral radius below one, the inverse sums successive rounds of future policy responses. At $M = 1$ the recursion has only the aggregate fringe state; there is no large-rival term, but future general-equilibrium feedback remains.

Proof of Proposition 4. At stationarity, the worker recursion implies $w_g - b - b_0 = d\mathcal{S}_g + \beta p \overline{\mathcal{S}}$. Subtracting the fringe equation gives the exact wage premium

$$w_i - w_f = \frac{\chi}{1 - \chi} s_i \frac{\eta\kappa}{q} d > 0. \quad (\text{G.11})$$

This compares employers within the same market and differs from the cross-market concentration comparisons. The level premium also differs from G_i , which measures the hiring incentive arising from the additional own-employment wage response.

G.3 Small employers and aggregate convergence

Fix M and write $n_i = \omega x_i$. Consider an interior equilibrium branch on which employment intensities remain positive and bounded, matching probabilities stay away from their boundaries, and wages satisfy the regular bargaining boundary condition. After removing the known within-firm production component, suppose policies, wage remainders, and values per unit of capacity vary smoothly with capacity and the state, with bounded mixed derivatives on a common neighborhood. At zero capacity, the aggregate fringe follows the atomistic equilibrium and a private employer has no effect on the aggregate state or another employer. Assume also that the difference in the full bargaining forcing from its atomistic counterpart is bounded by a constant times capacity along the own-employment path. These conditions specify the regular branch used for the following rates.

Proposition 5 (Individual and aggregate small-employer effects). *Under these conditions, at fixed M ,*

$$s_i = O(\omega), \quad \mathcal{H} = O(M\omega^2), \quad T_i, G_i, F_i = O(\omega), \quad R_i = O((M-1)\omega^2). \quad (\text{G.12})$$

With the common technology in (33),

$$x_i - x_f = O(\omega), \quad N - N^* = O(M\omega^2), \quad (\text{G.13})$$

$$w_i - w_f = O(\omega), \quad \bar{w} - w^* = O(M\omega^2), \quad (\text{G.14})$$

$$AN^\alpha - Y = O(M\omega^3), \quad (\text{G.15})$$

where $\bar{w}$ is the employment-weighted wage and (N^, w^*) is the atomistic steady state.*

Proof. Since $s_i = \omega x_i / N$, bounded positive intensities imply the share and HHI rates. Equation (35) gives the recruiting rate. For the wage remainder relative to the atomistic bargain, write $v_\omega + \chi x v'_\omega = e_\omega(x)$. The regular solution is

$$v_\omega(x) = \frac{1}{\chi} \int_0^1 t^{1/\chi-1} e_\omega(tx) dt.$$

The integration weights sum to one. The assumed uniform forcing bound therefore gives $v_\omega = O(\omega)$ and, at positive intensity, $v'_\omega = O(\omega)$. Since $n_i w_{i,n_i} = x_i w_{i,x_i}$, this proves the rate for the additional wage-bill term. The excluded homogeneous solution is proportional to $x^{-1/\chi}$ and violates zero-payroll regularity for $0 < \chi < 1$.

For a distinct large rival, smoothness and endpoint independence imply that its policy response to the original employer's state is $O(\omega)$, as is the cross-payoff term multiplying that

response. Their product is second order, giving the stated R_i rate after summing over rivals. The aggregate fringe policy has a bounded response in raw employment units and an $O(\omega)$ cross-payoff term, so $F_i = O(\omega)$.

The regular strategic and fringe wage formulas have the same limiting production component, whose derivative is bounded away from zero on the positive compact intensity interval. Together with (G.11), the wage-remainder bound implies $x_i - x_f = O(\omega)$. Aggregation gives $N - x_f = \omega \sum_i (x_i - x_f) = O(M\omega^2)$. The stationary fringe hiring condition is

$$0 = (1 - \chi) \left[\frac{\alpha A}{1 - \chi(1 - \alpha)} x_f^{\alpha-1} - b - b_0 \right] - d \frac{\kappa}{q} - \beta \chi \kappa \theta (1 + \eta \mathcal{H}). \quad (\text{G.16})$$

Replacing x_f by $N + O(M\omega^2)$ leaves the atomistic residual equal to $O(M\omega^2)$. Its derivative at the interior atomistic root is strictly negative, proving the employment rate. The fringe wage formula and the hiring-weighted large-employer wage premia give the mean-wage rate. Finally, expand production around common intensity N . Capacity-weighted first-order deviations sum to zero. The large-employer second-order terms have total weight $O(M\omega^3)$; the fringe term is $O(M^2\omega^4)$. Concavity makes the output loss nonnegative. $\square$

This fixed-count experiment shrinks the combined large-employer sector. The no-fringe limit in Proposition 1 instead increases the employer count while their total productive capacity remains one.

G.4 Efficiency and employment allocation

The planner equates marginal products across employer types, assigning equal employment per unit of productive capacity. Let $c = M\omega$ and $\ell = Ms$ denote the large sector's capacity and employment shares. Aggregate output is

$$Y = AN^\alpha [c^{1-\alpha} \ell^\alpha + (1 - c)^{1-\alpha} (1 - \ell)^\alpha] \leq AN^\alpha, \quad (\text{G.17})$$

with equality at $\ell = c$. Market power therefore changes both aggregate employment and its allocation across productive capacity.

The matching component of the private-planner gap for employer type g is

$$\mathcal{M}_g = \frac{\eta \kappa D}{\nu q} [\nu(1 - s_g) - \chi] + \beta \chi \eta \kappa \theta (s_g - \mathcal{H}), \quad (\text{G.18})$$

For an individual fringe employer, set $s_g = 0$. Hiring-weighted aggregation gives $\eta\kappa D[\nu(1 - \mathcal{H}) - \chi]/(\nu q)$, so $\chi = \nu(1 - \mathcal{H})$ removes the aggregate matching residual. Across employer types, however,

$$\mathcal{M}_i - \mathcal{M}_f = -s_i \frac{\eta\kappa}{q} [d + \beta(1 - \chi)p] < 0.$$

One common bargaining weight therefore does not remove both type-specific matching residuals. Full implementation also accounts for within-firm wage dilution, the GE wage-bill effect, future responses, and employment allocation. A common UI instrument similarly acts on both employer types; its welfare effect depends on the aggregate employment response and on how jobs move between them.

G.5 Matched-share comparisons

I compare one and two large employers at own hiring shares of 0.0095, 0.0886, and 0.4264 percent. For each comparison, capacity is adjusted to match a large employer’s equilibrium share across the two economies, using the same procedure as Table 6.

Table 28 reports the comparison at the largest matched share. Adding a second large employer changes the magnitude of the own-share recruiting contribution by only 0.054 percent and the GE wage-bill effect by 0.029 percent. Concentration in bargaining approximately doubles, while the additional large-rival response is small. The table separates this term from aggregate fringe adjustment.

Table 28: One and two large employers at a matched own hiring share

Euler contribution	One large employer	Two large employers
Own-share recruiting	−0.000845112	−0.000844655
Concentration in bargaining	−0.000003158	−0.000006312
GE wage-bill effect	−0.022429960	−0.022423466
Future fringe response	−0.000439946	−0.000440138
Future large-rival response	0	+0.000000416

Notes: Each large employer accounts for approximately 0.4264 percent of hiring. The capacity of a large employer is 0.005003746 with one large employer and 0.005 with two. Entries use the signed Euler units of Figure 4; negative values discourage hiring. The first two rows sum to T_i in (35). The standard Euler block, including within-firm Stole–Zwiebel bargaining, balances the contributions shown.

Increasing own size has a more pronounced effect on the relative importance of recruiting. With two large employers, the absolute recruiting contribution, including the concentration term, rises from 0.27 to 1.02 to 3.80 percent of the GE wage-bill effect across the three shares. These smaller-share comparisons extend the pattern in Figure 8.

The computations impose regular bargaining along the own-employment path and retain separate large-employer states. Maximum off-grid equation and Euler-accounting errors are 1.32×10^{-9} and 1.67×10^{-10} . All six stationary choices have negative own-action payoff curvature, and all tested small unilateral deviations reduce profits.

G.6 Equal-HHI comparisons

To vary the distribution of hiring while holding concentration fixed, I use three two-employer equilibria with capacities 0.009, 0.021, and 0.032 per employer. For each, I solve a one-large-employer economy, adjusting its capacity until its equilibrium share satisfies $s_1 = \sqrt{2}s_2$. No productivity, matching, bargaining, or policy parameter changes. The matched one-employer capacities are 0.01282985, 0.03004671, and 0.04597777.

Table 29: Equal concentration with one and two large employers

Hiring HHI	M	Own share (percent)	Combined share (percent)	Unemployment (percent)	Gap (basis points)
1.154	1	1.074	1.074	5.521720	0.444
1.154	2	0.760	1.519	5.523185	0.590
6.108	1	2.472	2.472	5.529546	1.226
6.108	2	1.748	3.495	5.533198	1.591
13.978	1	3.739	3.739	5.538312	2.103
13.978	2	2.644	5.287	5.543968	2.668

Notes: Hiring HHI is $10,000\mathcal{H}$; combined share is $M s$. The final column is unemployment above the atomistic benchmark of 5.517284 percent. Productive capacities match equilibrium HHI before rounding, with maximum absolute discrepancy below 6.9×10^{-13} on the unscaled HHI measure. All equilibria use the same baseline parameters and efficient aggregate production frontier.

The unemployment difference is 0.147, 0.365, and 0.566 basis point across the three pairs. Relative to the one-employer gap from the atomistic benchmark, these differences are 33.0, 29.8, and 26.9 percent. The channel accounts below trace the differences to hiring incentives.

In the Euler units of (G.8), the individual recruiting contribution at the largest HHI becomes less contractionary, changing from -0.007599 to -0.005433 when one large employer is replaced by two smaller ones. The aggregate recruiting contribution is almost unchanged: -0.0005161 versus -0.0005150 . The individual GE wage-bill effect changes much less, from -0.02682 to -0.02585 . The increase in the combined large-employer share makes its aggregate contribution more contractionary, from -0.001003 to -0.001367 . The aggregate fringe-response contribution changes from $+0.0000787$ to $+0.0000496$, and the large-rival term contributes $+0.0000036$ with two employers.

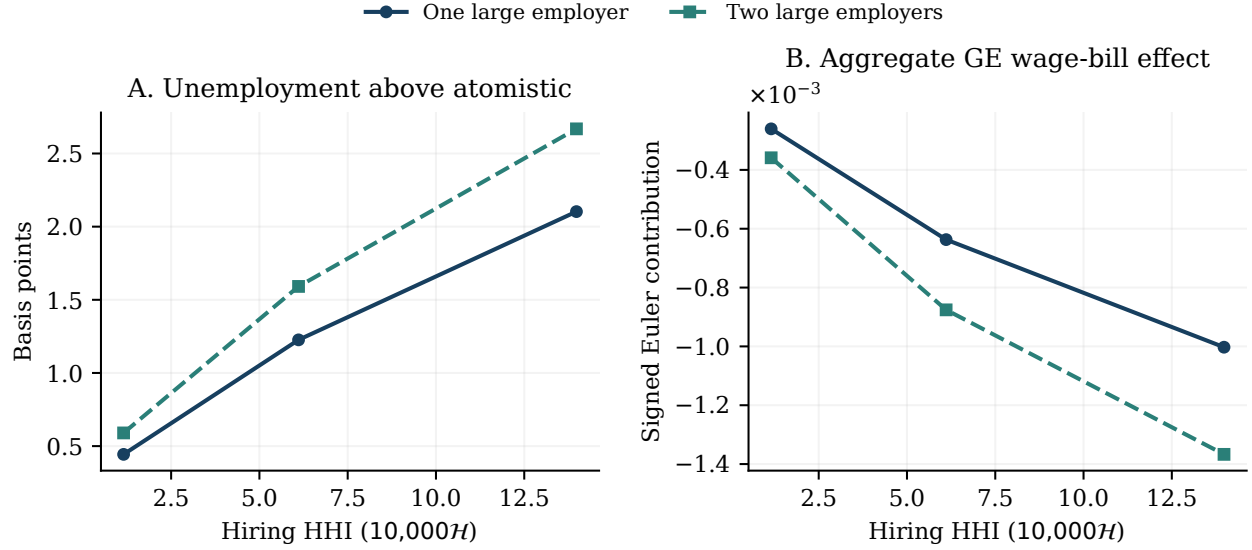


Figure 19: Unemployment and wage-bill exposure at equal HHI

Notes: Each pair matches equilibrium HHI by changing productive capacity. The left panel plots unemployment above the atomistic benchmark. The right reports the signed aggregate wage-bill contribution $\sum_i s_i G_i = MsG_i$ in the Euler units of Figure 4. Negative values discourage hiring. Points are solved equilibria; lines connect the three comparisons.

G.7 Numerical implementation and verification

The fringe calculations solve the regular bargaining equation along the own-employment path jointly with continuation values and hiring policies. The known within-firm production component is removed before approximating the wage remainder. Integrating from zero employment imposes zero-payroll regularity and excludes the homogeneous component left unrestricted by the differential equation at the stationary point. This selects the regular wage schedule and its employment derivatives.

Employer symmetry reduces repeated coefficients without collapsing distinct employment states. Two-large-employer solutions retain both individual employment coordinates and aggregate fringe employment. Independent checks reconstruct the firm and worker Bellman equations, the unemployment value, primitive wage bargaining, and hiring conditions at fresh states. They also check the wage-premium identity, recursive cross-envelope account, and within-domain unilateral deviations.

For the equal-HHI exercise, all seven checkpoints—the three one-employer matches, three two-employer targets, and a refined one-employer solution—pass these tests. The largest independent primitive residual is 2.70×10^{-8} . Raising the approximation degree for the largest one-employer match from 29 to 33 changes unemployment by less than 8.0×10^{-11} .

percentage point and each individual Euler contribution by less than 4.3×10^{-11} . The full aggregate Euler account closes to less than 10^{-7} at every reported match.

Worker income in these linear-baseline exercises is wages plus nonemployment value, excluding profits. Adding workers' share of profits gives the corresponding incidence measure.